Finite-Spin Ward-Stress Gauss-Law Audit for the Chiral Spin-2 Carrier

QATNU technical audit

July 2, 2026

Abstract

We test the next microscopic step for the Ward-completed chiral spin-2 carrier: replacing independent TT source coordinates by a full local symmetric stress tensor S_{ij} subject to a local Ward/Gauss law $\nabla_i S_{ij} = 0$. At harmonic level the local covariant square $\|S_{ij} - \alpha h_{ij}^{\mathrm{wall}}\|^2$ is reduced to the exact Gauss surface. The result is positive as a finite-spin/compact-rotor Gauss-law target: the constraint is local and integer, random constrained stress has order-one TT residue, the coupled model has exactly two soft TT branches, $z \simeq 1$, and $\chi_S(0,k) \sim k^{-2}$. Shift-preserving perturbations keep the pole, while shift-breaking stress or wall terms gap it. This is still not a complete nonperturbative finite-d phase proof; it is the harmonic and constraint-rank audit of the full stress Gauss law.

1 Local stress Gauss law

The proposed compact stress sector has one symmetric stress rotor per tensor component and site. The microscopic Ward law is

$$G_j(x) = \sum_i [S_{ij}(x) - S_{ij}(x - \hat{e}_i)] = 0. \tag{1}$$

The coefficients are only $0, \pm 1$, so this route has no rational charge hierarchy. In Fourier space, $G_j(k) = d_i(k) S_{ij}(k)$, and the divergence matrix has rank three for every nonzero momentum. Hence the divergence-free stress space has dimension three; inside it the TT subspace has dimension two.

For the tested momenta, the maximum Ward residuals were

$$\|DN\|_\infty \le 7.482e-17, \quad \|DT_{\mathrm{TT}}\|_\infty \le 1.071e-16, \quad \|\mathrm{tr} T_{\mathrm{TT}}\|_\infty \le 4.441e-16. \tag{2}$$

A random divergence-free stress has mean TT weight 0.664049, close to the expected 2/3. Thus the Gauss law supplies finite TT source residue without building the source as a Beltrami or Cotton derivative.

2 Ward-completed square on the Gauss surface

The local square is

$$H_m = \frac{\Omega^2}{2} \left\| S_{ij} - \alpha h_{ij}^{\mathrm{wall}} \right\|^2. \tag{3}$$

In the physical quotient h_{ij}^{wall} has the two TT components of the chiral boundary carrier. This expression is local in tensor components before quotienting; the TT basis is used only for symbol reduction.

At the origin there are exactly two diagonal shift zero modes. Their residuals are

$$\|K(0) z_1\| = 1.388e-17, \qquad \|K(0) z_2\| = 3.103e-17. \tag{4}$$

The third and fourth gaps at the origin are

$$\Delta_3(0) = 0.184537, \qquad \Delta_4(0) = 0.184537. \tag{5}$$

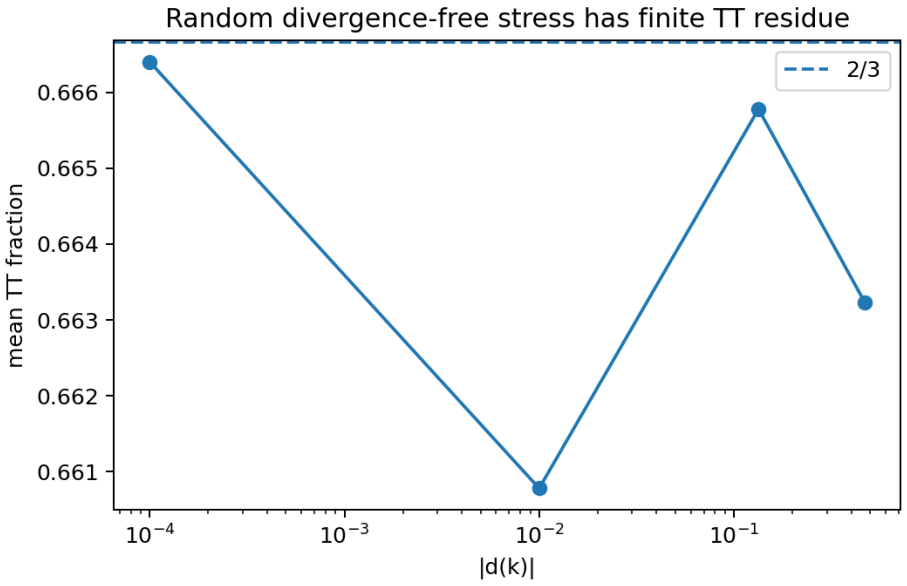

Figure 1: A random stress satisfying $d_i S_{ij} = 0$ has order-one TT content.

The infrared fits give

$$\omega_1 \sim |k|^{1.000011335}, \qquad \omega_2 \sim |k|^{0.999997506}, \tag{6}$$

with Ward-source response

$$\chi_1(0,k) \sim |k|^{-2.000086438}, \qquad \chi_2(0,k) \sim |k|^{-2.000086559}. \tag{7}$$

At $|k| = 10^{-3}$, exactly 2 modes lie below the 0.1 threshold.

3 Finite periodic Gauss-rank count

On an L^3 periodic lattice, the Fourier-symbol rank audit gives

L	V	computed rank	$3V-3$	physical stress coordinates
4	64	189	189	195
6	216	645	645	651
8	512	1533	1533	1539
10	1000	2997	2997	3003
12	1728	5181	5181	5187

The exact count is $3(V-1)$ independent Gauss constraints and $3V+3$ stress coordinates after the Ward constraint. The extra $+3$ are the global zero-momentum stresses, which are then handled by the local Ward square and diagonal shift structure.

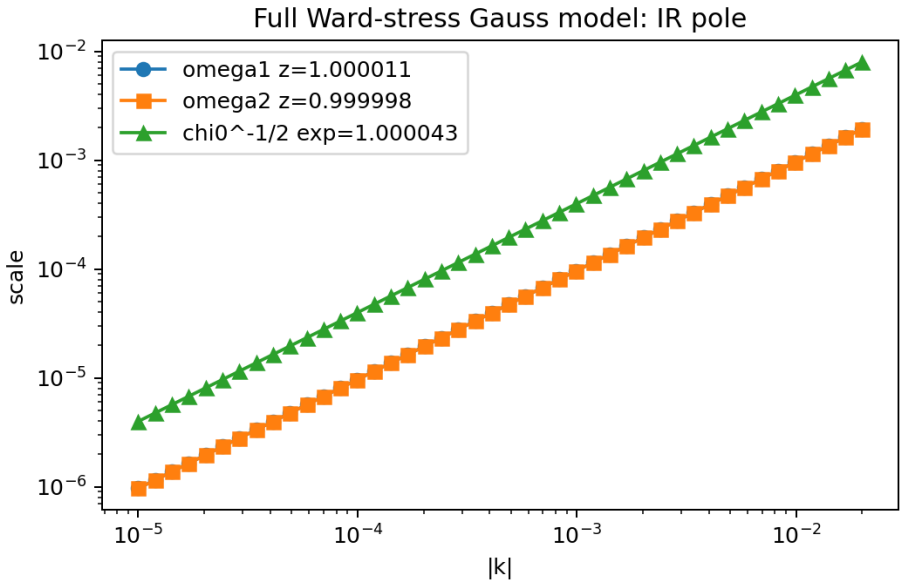

Figure 2: The full stress Gauss-law model keeps the two linear TT branches and the k^{-2} Ward-source response.

4 Finite-box branch screen

L	$\omega_1^{\min}$	$\omega_2^{\min}$	third branch	extra soft points
6	0.186602	0.186602	0.383759	0
8	0.108321	0.108321	0.279003	0
10	0.0748908	0.0748908	0.234809	0
12	0.0577083	0.0577083	0.214113	0
14	0.047383	0.047383	0.203447	0
16	0.0404346	0.0404346	0.197465	0
20	0.0315221	0.0315221	0.191548	0
24	0.0259484	0.0259484	0.188901	0

Using the asymptotic axis-momentum fit for $L \geq 32$, the lowest finite-box frequency scales as

$$\omega_{\min} \sim k_{\min}^{1.007796593}. \tag{8}$$

No point in the audited finite boxes has more than two modes below the soft threshold.

5 Perturbations

Random positive perturbations constrained to preserve both diagonal shifts passed 64/64 tests. The fitted ranges were

$$z \in [0.999986, 1.000084], \quad \chi \text{ exponent} \in [-2.000013, -2.000004]. \tag{9}$$

Generic local stress or wall terms that break the diagonal shift opened gaps in the range

$$\Delta \in [4.838e-06, 4.363e-02], \tag{10}$$

with gap-amplitude exponent 0.522529. This is the expected square-root mass relevance.

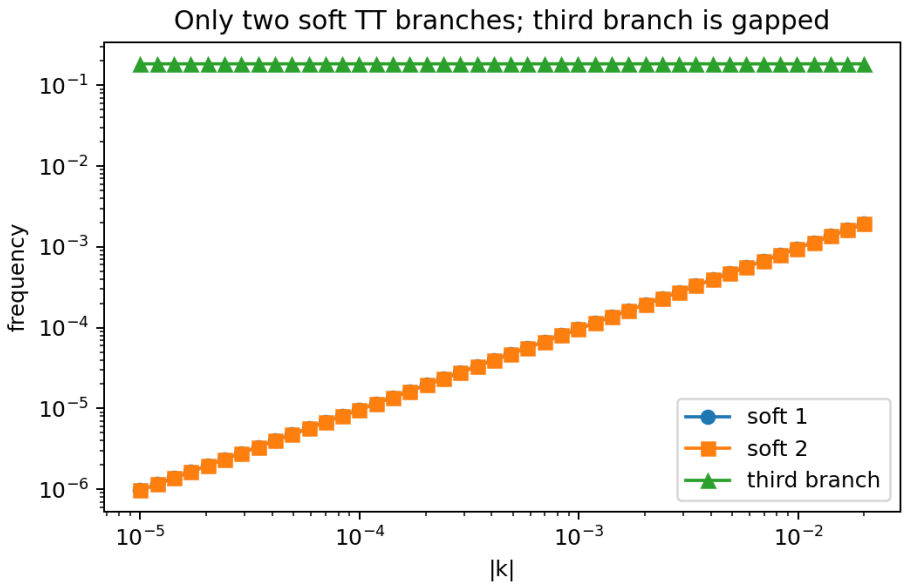

Figure 3: The third branch stays gapped while the two TT branches soften linearly.

6 Verdict

The microscopic Ward-stress Gauss law passes the harmonic and rank-level tests:

- the Gauss law is local, integer, and charge-hierarchy-free;
- constrained stresses retain order-one TT residue;
- the Ward-completed square gives two and only two linear TT branches;
- the stress response scales as k^{-2};
- shift-preserving perturbations keep the pole, while shift-breaking terms gap it.

This advances the finite-spin route from a diagonal $U(1)$ phase screen to a full stress-Gauss harmonic target. It is not yet a nonperturbative finite-d proof. The remaining obligations are to simulate or prove the full compact finite-spin tensor/matter model with the Gauss law imposed microscopically, to control compact defects and anisotropies, and to verify that no extra Goldstone or mirror sector appears beyond the harmonic quotient.

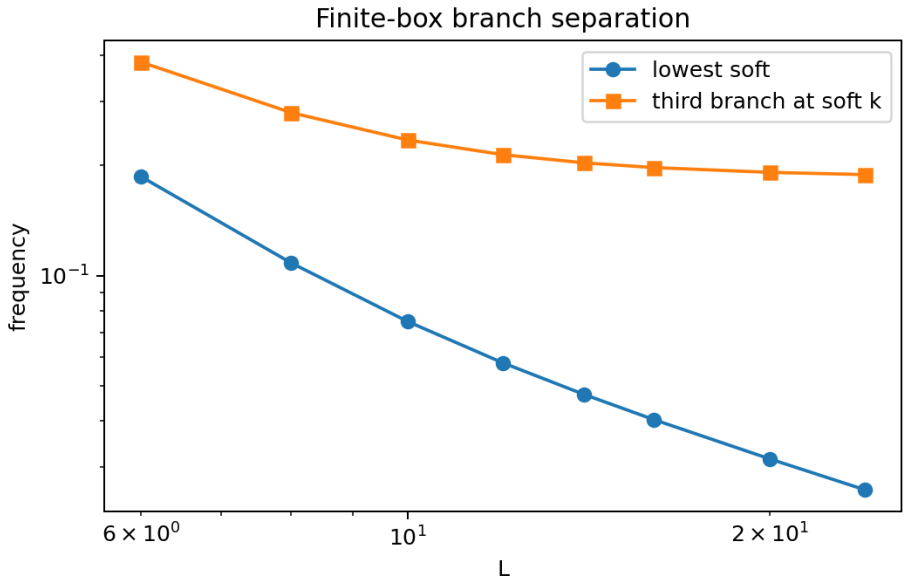

Figure 4: Finite-box branch separation: the first two branches soften, while the third remains gapped.

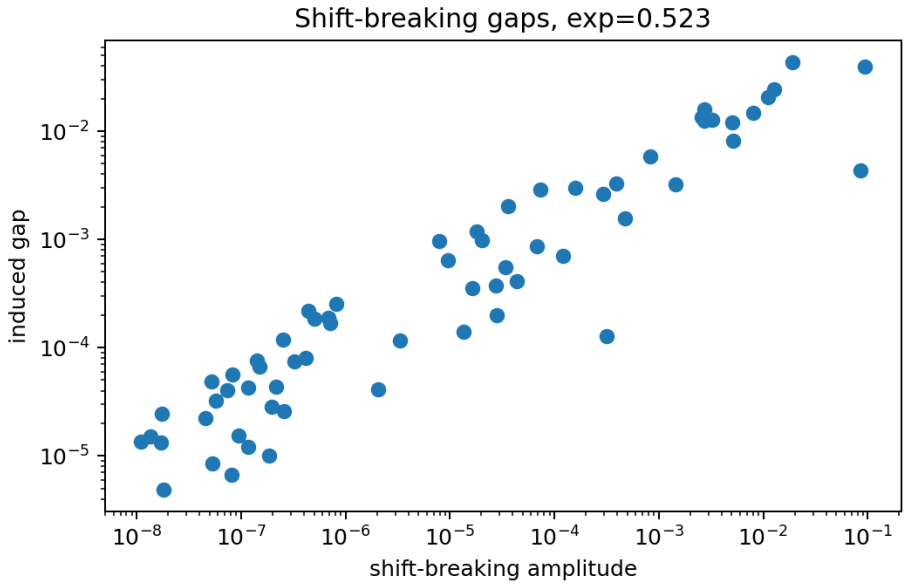

Figure 5: Shift-breaking local stress/wall terms gap the pole.