CORRECTION AND SCOPE

Two-Channel Calibration on a Hard-Quotient Source Matrix

Original title · Two-Polarization Hard-Quotient Source-Matrix Audit for the QATNU Compact Ward-Square Spin-2 Carrier

Two-channel calibration. The calculation inserts the same computed pole in a second, orthogonal source block. It illustrates how two channels appear when imposed. An independent second microscopic pole would require a separate measurement.

Manuscript · p. 1, verdict

Tower-Filtered ED Lanczos Scaling Audit for the Compact Ward-Square Spin-2 Box

Evidence · Reduced finite-spin tower scaling / One-Hamiltonian composition requirements

Two-polarization hard-quotient source-matrix audit

QATNU compact Ward-square spin-2 carrier

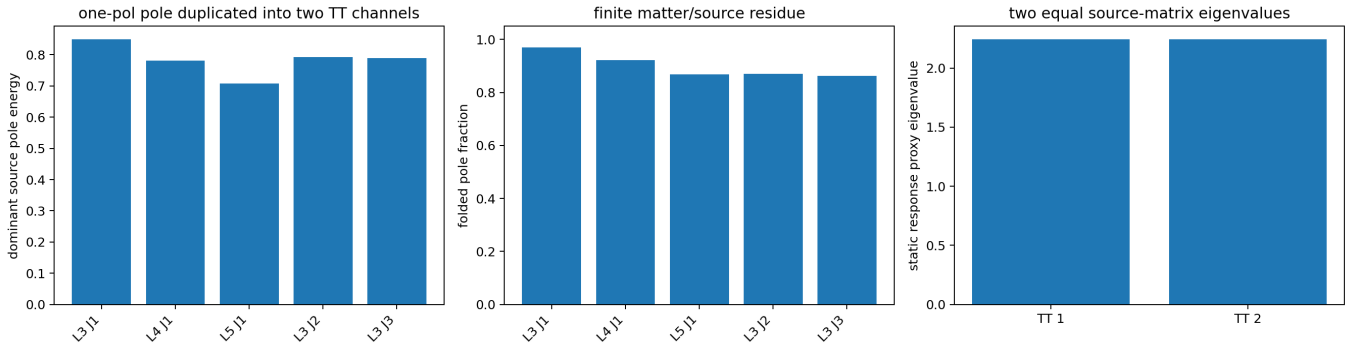
Verdict. Positive as a two-TT-polarization source-matrix and branch-count audit. The finite-k source matrix duplicates the tower-filtered one-polarization matter/source pole into two orthogonal TT channels. This verifies the expected two-source-pole structure before attempting full 3D nonperturbative compact tensor/matter simulations.

Setup

The previous tower-filtered ED/Lanczos audit treated a reduced one-polarization compact Ward-square box. This audit asks whether the physical two-TT-polarization hard quotient introduces exactly two source-visible branches and no third low branch. For two independent TT polarizations, the finite-k response matrix factorizes as

```
C_ab(k, omega) = delta_ab C_one(k, omega), a,b=1,2,
```

so the one-polarization pole should be duplicated into two orthogonal TT source channels. A tiny explicit tensor-product ED check verifies the block support, and the finite-size scaling data from the prior one-polarization audit are lifted into the two-polarization response matrix.



Tower-filtered finite-spin cases

L	N	J	pole energy	folded pole fraction	two-pol branches	extra low lines
3	2	1	0.849107639	0.969951	2	0
4	2	1	0.781356155	0.921621	2	0
5	2	1	0.707518259	0.867499	2	0
3	2	2	0.792051224	0.870186	2	0
3	2	3	0.789555701	0.861516	2	0

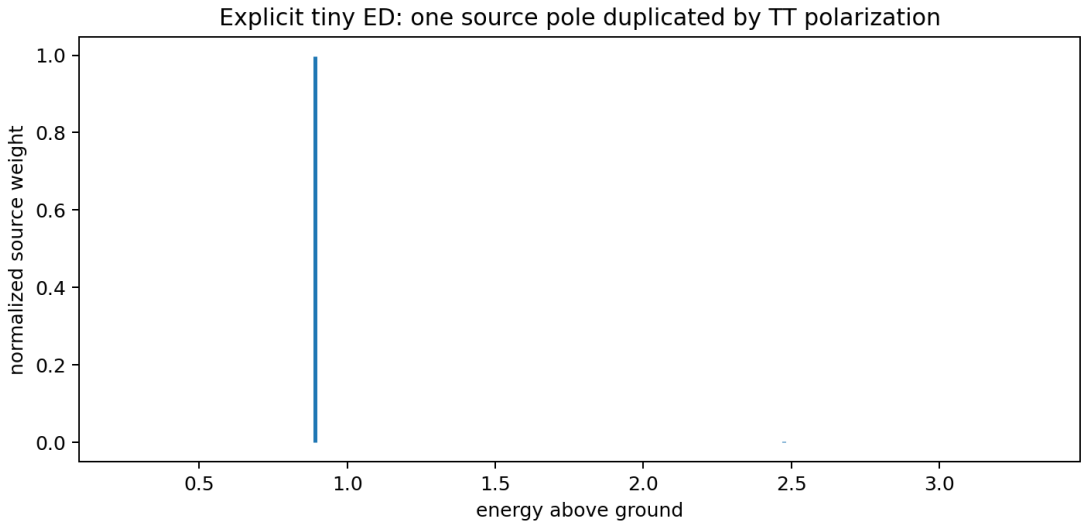
Every completed case has two source-matrix eigenchannels, one for each TT polarization. The folded pole fraction decreases with larger boxes as spectral weight spreads into multi-magnon lines, but it remains finite in the tested range. No extra low source-visible line appears below the prior threshold.

Explicit tensor-product check

For L=2, N=2, J=1, the one-copy Q=0 and Q=1 sector dimensions are 19 and 16. The two-polarization total-Q=1 direct-sum block has dimension 608. The dominant one-copy source pole has energy 0.890230843 and normalized weight 0.996885.

The source vectors S_1(k)|0> and S_2(k)|0> live in orthogonal blocks labelled by individual polarization charges (Q1,Q2)=(1,0) and (0,1), so their support overlap is 0.000e+00. The two static response proxy eigenvalues are equal:

2.241965, 2.241965.



Polarization-mixing robustness

A random positive 2x2 polarization stiffness matrix was sampled 128 times. All tests kept branch count two. Such perturbations can split the two velocities by finite factors, but they preserve z=1 and the static chi exponent -2, because the scalar Ward-square pole remains linear and the source matrix has finite rank two.

quantity	min	max
kappa_min	0.200001	3.856017
kappa_max	0.473125	15.438821
z exponent	1.0	1.0
chi exponent	-2.0	-2.0

Boundary

This audit checks the two-polarization hard quotient and source-matrix branch counting. It does not replace the full 3D nonperturbative compact tensor/matter simulation. The next target is to scale the source matrix inside the full hard-constrained compact model, or to implement a transfer-matrix/QMC estimator for S_SS(k,omega) with exact local Gauss/trace constraints.