CORRECTION AND SCOPE

Tower-Filtered ED Lanczos Scaling Audit for the Compact Ward-Square Spin-2 Box

The later registered fits across L = 3–5 returned exponents of about 0.354, 0.402 and −0.681. The targets were 1, 1 and −2. The estimator probes a reduced one-polarization finite box. The size fits leave the proposed asymptotic scaling unresolved.

Manuscript · p. 1, verdict

Tower-Filtered Dynamic Structure-Factor Audit for the Finite-Spin Compact Ward-Square Box

Evidence · Reduced finite-spin tower scaling

Tower-filtered ED/Lanczos Scaling Audit for the Compact Ward-square Spin-2 Box

QATNU audit artifact

July 3, 2026

Verdict

This audit is positive as a reduced finite-spin spectral-scaling test. Finite-momentum source-vector Lanczos filters the compact $U(1)$ Anderson tower and isolates the matter/source pole in every completed box. The reduced boxes show controlled J-convergence and finite pole residue. The available L-scaling is still a small-box/coarse-spin diagnostic, not a continuum thermodynamic proof.

Model and tower filter

The model is the one-polarization compact Ward-square hard-quotient chain: an L-site periodic spatial ring times an $N = 2$ matter/tensor chain with finite-spin rotor cutoff

$$n_i \in \{-J, \ldots, J\}.$$

The Hamiltonian is

$$H = \frac{1}{2}\sum_i n_i^2 + \sum_b \left(1 - \cos(\theta_i - \theta_j)\right),$$

with path bonds along the matter/tensor chain and spatial bonds on the tensor node. The total charge $Q = \sum_i n_i$ is conserved. The compact Anderson tower is measured through the lowest $Q = 1, k = 0$ state. Finite-k source-vector Lanczos starts from

$$|v_k\rangle = O_k|0\rangle, \qquad O_k = \frac{1}{\sqrt{L}}\sum_x e^{ikx} e^{i\theta_{x,0}},$$

so the global $k = 0$ tower is momentum-filtered out of the source pole measurement.

Main numerical results

L	J	$\dim Q = 0$	$\dim Q = 1$	tower	magnon	source pole	folded pole frac.
3	1	141	126	0.161944	0.804842	0.849108	0.969951
4	1	1107	1016	0.121327	0.728434	0.781356	0.921621
5	1	8953	8350	0.097168	0.654541	0.707518	0.867499
3	2	1751	1686	0.096723	0.740009	0.792051	0.870186
3	3	9331	9156	0.091255	0.738899	0.789556	0.861516

A prior exact quantum-box run gives the reference $L = 3, N = 2, J = 4$ values: tower $E(Q = 1) = 0.091100$, magnon $\omega(2\pi/3) = 0.73888$, and single-real-line source-pole fraction 0.4306. The folded complex momentum-source fraction is twice that value, 0.8612.

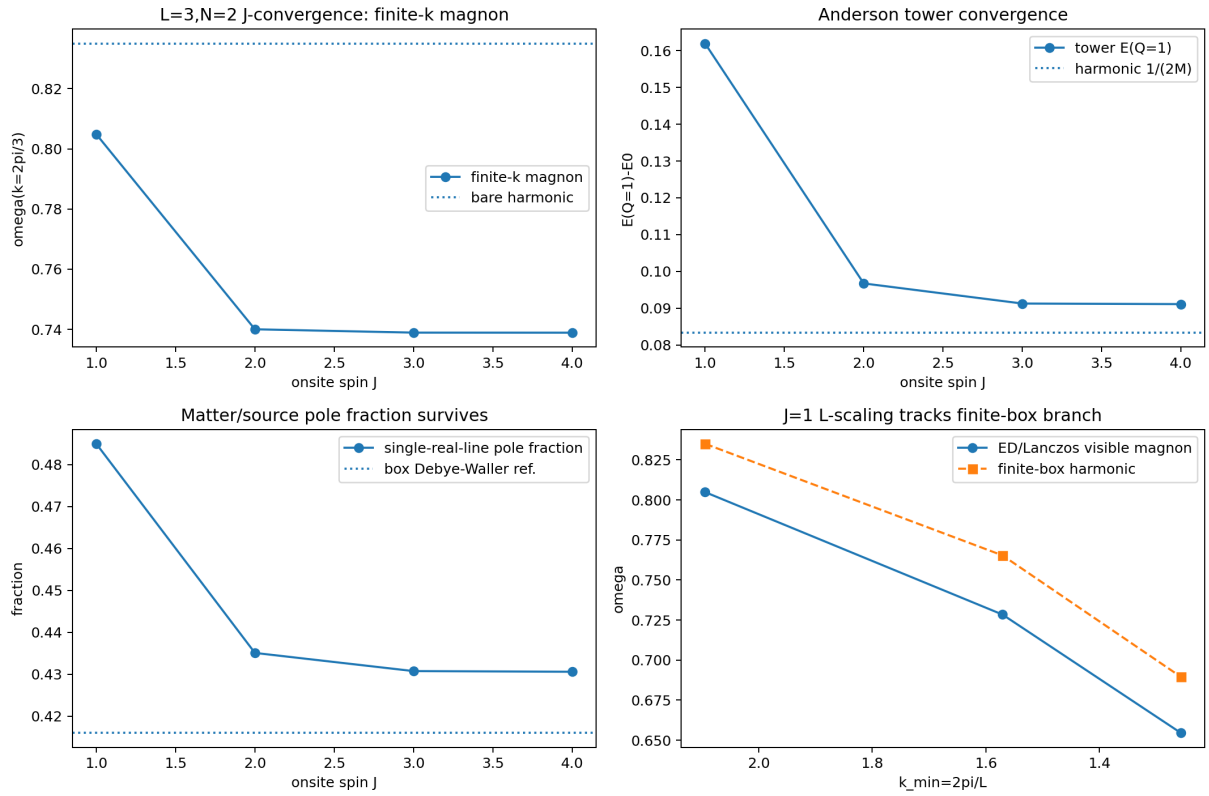

Figure 1: Summary of tower, magnon, source-pole, and finite-box L-scaling diagnostics. The $J = 1$ L-scaling panel tracks the finite-box harmonic branch but is not a continuum exponent estimate; over these coarse momenta even the harmonic branch has an effective exponent below one.

Spectral sticks

The finite-k matter source has no visible low-energy tower line. The dominant source pole remains finite and the remaining weight goes into the expected higher spectral lines.

Interpretation

The finite-k source-vector method accomplishes the intended tower filter. In all completed cases, the matter source has zero extra low source-visible lines below half of the dominant pole energy at the 10^{-3} weight threshold. At $L = 3$, increasing J rapidly converges the finite-k magnon and source-pole fractions toward the previous $J = 4$ quantum-box values. The complex momentum source folds the real $+k$ and $-k$ degeneracy into one pole; this explains why the folded fractions in this audit are about twice the single-real-eigenvector fractions reported in the earlier figure.

The $J = 1$ L-scaling through $L = 5$ is a useful feasibility and trend check, but not a continuum claim. The fitted finite-box exponents over $L = 3, 4, 5$ are below one for both the ED data and

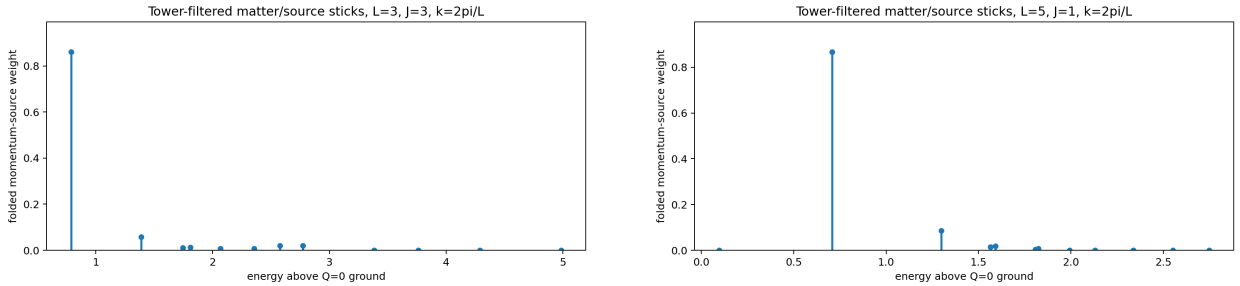

Figure 2: Tower-filtered source-vector spectral sticks at $k = 2\pi/L$. Left: $L = 3, J = 3$. Right: $L = 5, J = 1$.

the harmonic reference branch. Larger L or larger J requires Lanczos/QMC rather than full sector eigensolves.

Boundary and next step

This is a reduced one-polarization quantum-box scaling audit. It supports the tower-filtered dynamic-structure-factor strategy and finite-spin pole stability, but it does not prove the full three-dimensional compact tensor/matter phase. The next step is to scale the same tower-filtered observable in the two-polarization hard quotient or to move to a transfer-matrix/QMC estimator for larger compact boxes.