Full Tensor-Quotient Seagull Locality Audit

QATNU computational audit

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Abstract

We test the next microscopic obligation after the finite-spin Ward-stress Gauss-law audit: whether the Ward-completed square can be promoted from an already-reduced TT label to a full local symmetric tensor variable while preserving the first-class tensor quotient. The local stress Gauss law can be supplemented by a local trace constraint, producing an independent finite-residue TT stress source with integer nearest-neighbor constraints. The source cross-term is then local and first-class. The obstruction moves to the symmetry-fixed h^2 seagull: the local full tensor seagull and local trace-free seagull fail the first-class normalizer condition, while the exact first-class seagull is the TT projector and has an algebraic nonlocal tail. Thus the Ward-stress source is microscopic and local, but a full local microscopic completion must generate the seagull through an enlarged gauge/matter symmetry rather than insert a local $h_{ij}h_{ij}$ term.

1 Question

The effective Ward-completed matter square is

$$H_m = \frac{\Omega^2}{2}\|S - \alpha h_{\rm wall}\|^2. \tag{1}$$

In the reduced carrier this is a square over the two TT labels. A microscopic finite-spin implementation must instead start with local symmetric tensor variables A_{ij} and local stress variables S_{ij}. The audit separates the square into its source cross-term and its required seagull.

The tensor first-class constraints are represented at symbol level by

$$T = \mathrm{tr}A + \phi, \qquad D = d_i d_j A_{ij}, \qquad L_j = d_i B_{ij} - d_j \mathrm{tr}B + 2d_j\pi. \tag{2}$$

Their first-class residual was

$$\|QP^T\|_\infty = 2.774e-17. \tag{3}$$

2 Local Ward stress

A full local symmetric stress with only $\nabla_i S_{ij} = 0$ has a finite TT component but also a scalar trace component. Adding the local constraint $\mathrm{tr}S = 0$ gives an exact local TT source variable without derivative suppression:

$$\nabla_i S_{ij} = 0, \qquad \mathrm{tr}S = 0. \tag{4}$$

For nonzero momentum this leaves two stress directions. On finite periodic boxes the integer nearest-neighbor constraint ranks obey

$$\mathrm{rank}(\nabla\cdot S) = 3(V-1), \qquad \mathrm{rank}(\nabla\cdot S, \mathrm{tr}S) = 4V-3. \tag{5}$$

The latter leaves $2V+3$ coordinate degrees of freedom, i.e. two local stress polarizations for each nonzero momentum plus global uniform trace-free modes.

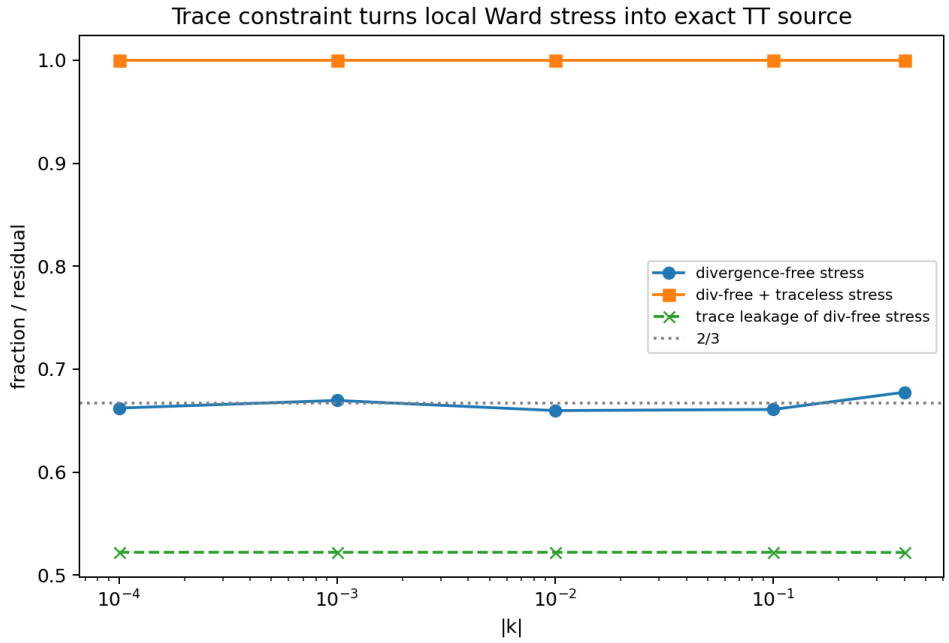

Figure 1: A divergence-free stress has mean TT fraction near 2/3 and scalar trace leakage. The additional local trace constraint makes the stress exactly TT at nonzero momentum.

3 The seagull test

The cross-term $S_{ij}A_{ij}$ is first-class if S is divergence-free and traceless: it annihilates vector-gauge and trace directions. The seagull is harder. For a potential K on A_{ij}, preservation of the momentum constraints requires

$$PKN_Q = 0, \tag{6}$$

where N_Q spans the configuration constraint surface. We tested five seagull symbols: full local identity, local trace-free projector, divergence-free projector, exact TT projector, and Cotton-order TT character.

The maximum residuals over the sampled ray were

seagull	max residual	residual exponent
full local I	9.297e-01	1.000
local tracefree	8.567e-01	1.000
divfree projected	5.230e-01	1.000
exact TT projected	5.034e-16	–
cotton order TT	2.260e-17	–

4 Projector nonlocality

The exact TT seagull is first-class, but it is not a bounded-range microscopic operator. On an $L = 24$ Fourier grid, the representative kernel component has shell-mean tail exponent

$$|K_{\mathrm{TT}}(r)| \sim r^{-2.583}. \tag{7}$$

This is the same algebraic locality obstruction seen in earlier projector audits.

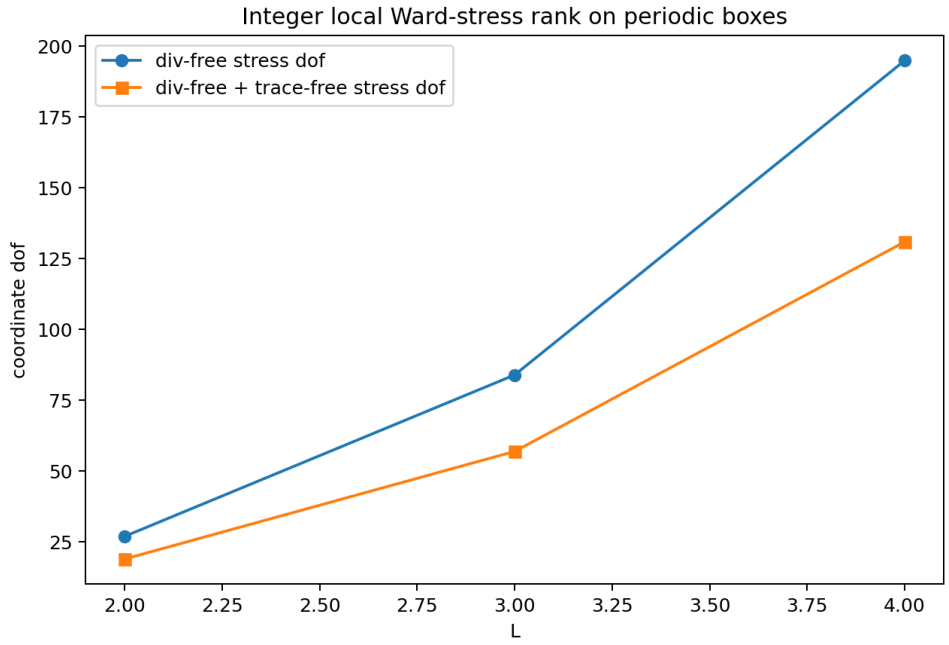

Figure 2: Finite periodic lattice rank counts for the local integer stress constraints.

5 Verdict

The microscopic stress side is positive: a local integer Gauss law plus local trace constraint gives an exact independent TT stress source with finite residue. The local source cross-term is compatible with the first-class tensor quotient. The obstruction is the seagull. A local $A_{ij}A_{ij}$ seagull or local trace-free seagull breaks the first-class tensor constraints, while the first-class order-one seagull is the nonlocal TT projector. Therefore the full finite-spin completion cannot simply use a local tensor square. It must generate the seagull through a larger local gauge/matter Ward symmetry, or accept an exact Dirac/projected implementation.

The next calculation should test an enlarged compact matter-gauge model in which the stress variables transform with the tensor gauge symmetry so that the whole covariant square is local and first-class before reduction.

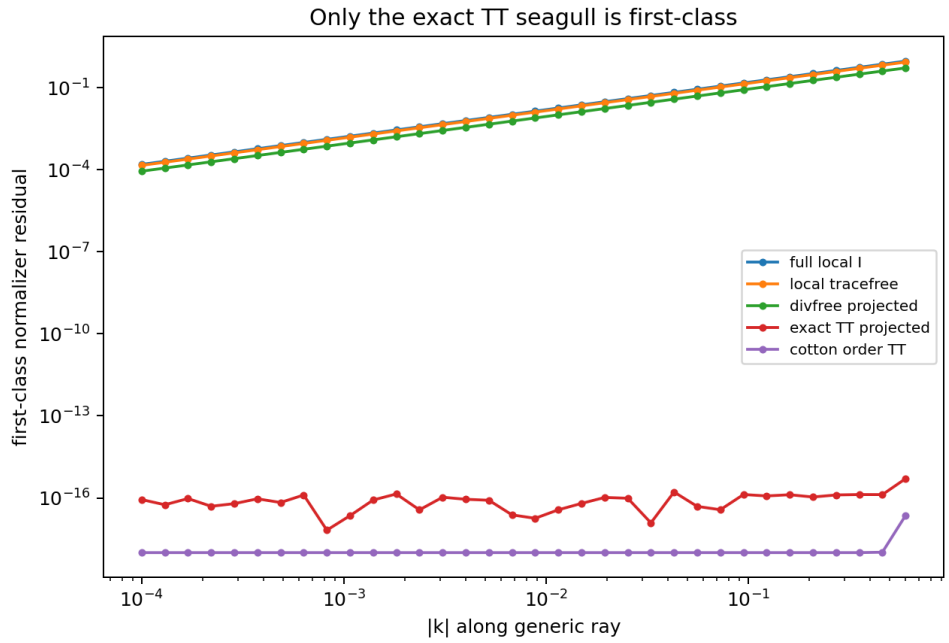

Figure 3: First-class normalizer residuals. Local full and trace-free seagulls fail. The exact TT seagull passes but is a projector.

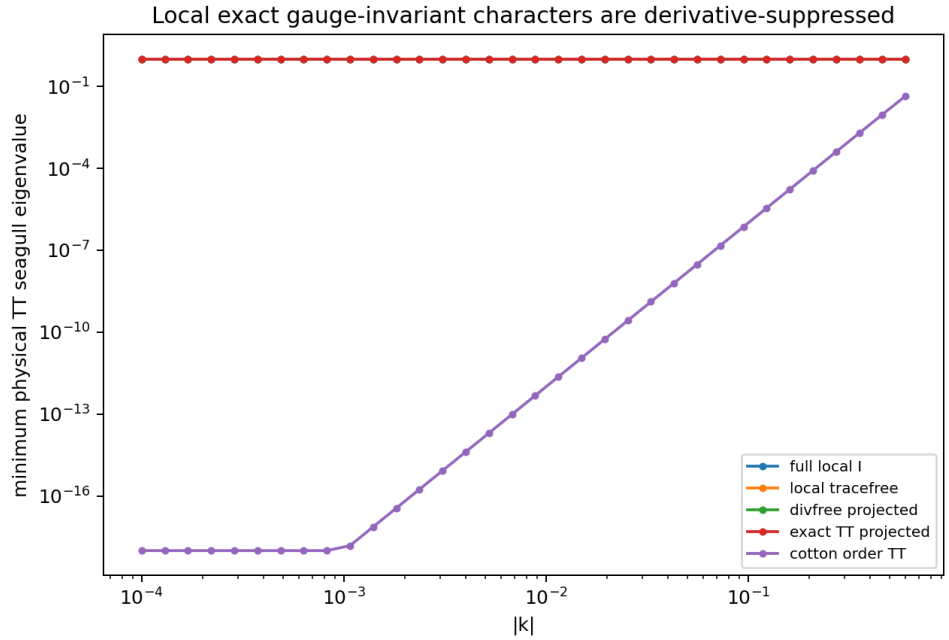

Figure 4: Physical TT eigenvalues. Exact local gauge-invariant characters can be first-class only with derivative suppression, while an order-one seagull requires the TT projector.

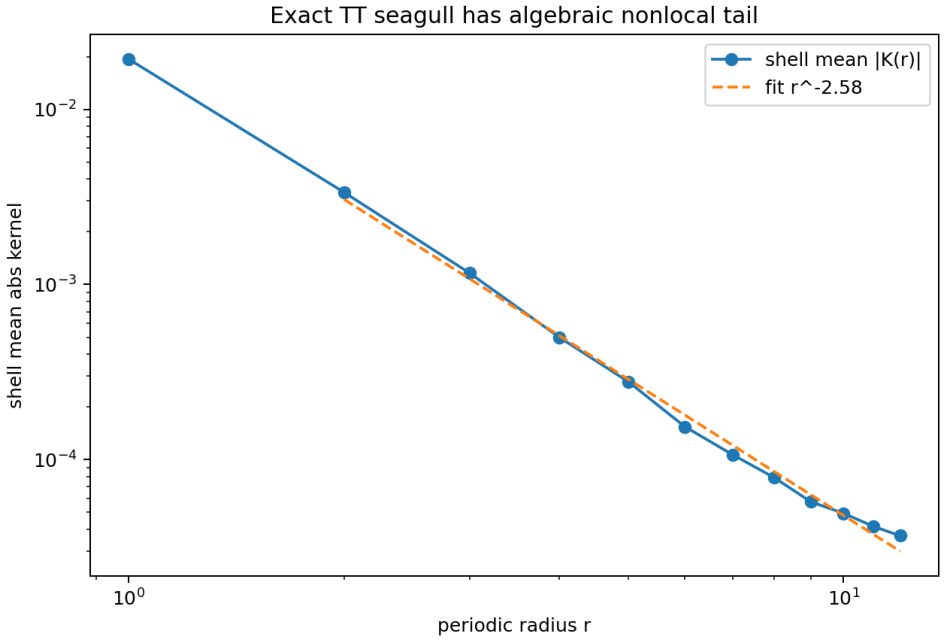

Figure 5: Real-space tail of a representative exact TT projector kernel.