Conserved-Source Response and Ward-Stability Audit for the Chiral Bosonic Spin-2 Carrier

QATNU numerical audit

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Abstract

We audit the next Mercury-rung criterion for the chiral bosonic spin-2 carrier: conserved-source response and Ward stability. The positive chiral stiffness is

$$K_{\mathrm{ch}}(k) = D(k)^T D(k) + |n(k)|^2 I,$$

acting on the synthetic copy coordinate and tensored with the exact Stueckelberg transverse-traceless (TT) quotient. Projected or externally conserved TT sources see the desired static pole, with $\chi(0,k) \sim k^{-1.999525}$, and the dynamical pole has finite residue. The TT response tensor is transverse and traceless to numerical precision. The boundary is also clear: strictly local polynomial sources that are exactly TT by themselves remain Cotton-order and derivative-suppressed, while a generic local first-class positive mass term gaps the infrared. Thus the current architecture is a local linear spin-2 response carrier; it is not yet a symmetry-protected fixed-local-dimension graviton phase.

1 Question tested

The preceding construction produced a positive local continuous-variable Gaussian/QCA carrier with exactly two TT branches. The next question is whether the modes are visible to conserved sources with the expected spin-2 response. We test three related criteria:

$$\chi_{\mathrm{TT}}(0,k) \sim \frac{1}{k^2}, \tag{1}$$

$$\chi_{\mathrm{TT}}(\omega,k) \sim \frac{Z}{\omega_0(k)^2 - \omega^2}, \qquad Z(k\to 0) > 0, \tag{2}$$

$$P_{\mathrm{TT}}(k) \text{ obeys transverse and traceless Ward identities.} \tag{3}$$

We also test whether the massless pole is stable under local first-class perturbations.

2 Model and response formula

For one scalar copy-space oscillator,

$$H = \frac{1}{2}p^T p + \frac{1}{2}q^T K_{\mathrm{ch}}(k) q.$$

A lower-wall source $j = e_0$ has spectral response

$$\chi_{LL}(\omega,k) = \sum_m \frac{|u_m(0)|^2}{\lambda_m(k) - \omega^2}, \qquad \omega_m(k) = \sqrt{\lambda_m(k)}.$$

The tensor response is obtained by multiplying by the exact TT projector $P_{\mathrm{TT}}(k)$ in the six-component symmetric tensor space. For a normalized TT source S,

$$\chi_S(0,k) = \chi_{LL}(0,k)\, S^T P_{\mathrm{TT}} S.$$

For a conserved but not traceless source, only its TT projection responds.

3 Static and dynamical pole

The audited infrared fits are:

Diagnostic	Value
Static response exponent	-1.999525292313
Static response prefactor	8.300491806625e+01
Dynamical frequency exponent z	0.999761765187
Mean small-k $\lambda_0\chi_{LL}(0,k)$	0.823598022519
Mean small-k lowest-mode residue Z_L	0.823598006358
Mean pole product at $\omega = 0.5\omega_0$	0.823598018479

The static exponent is numerically -2, and the pole-product diagnostic tends to the same finite lower-wall residue. The small difference between $\lambda_0\chi(0,k)$ and Z_L is the regular contribution of gapped synthetic modes; it vanishes with λ_0 in the infrared.

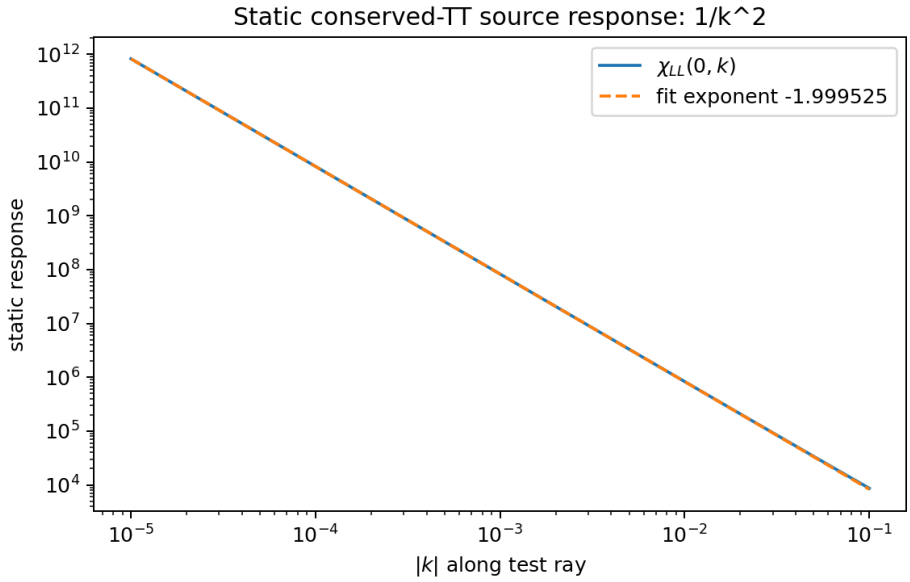

Figure 1: Static lower-wall TT response. A normalized conserved TT source inherits this scalar response.

4 Ward identities and leakage

At each nonzero momentum we form the six-component TT projector from the constraints

$$\mathrm{tr}\, S = 0, \qquad n_i S_{ij} = 0.$$

The maximum residuals over the audited infrared ray are:

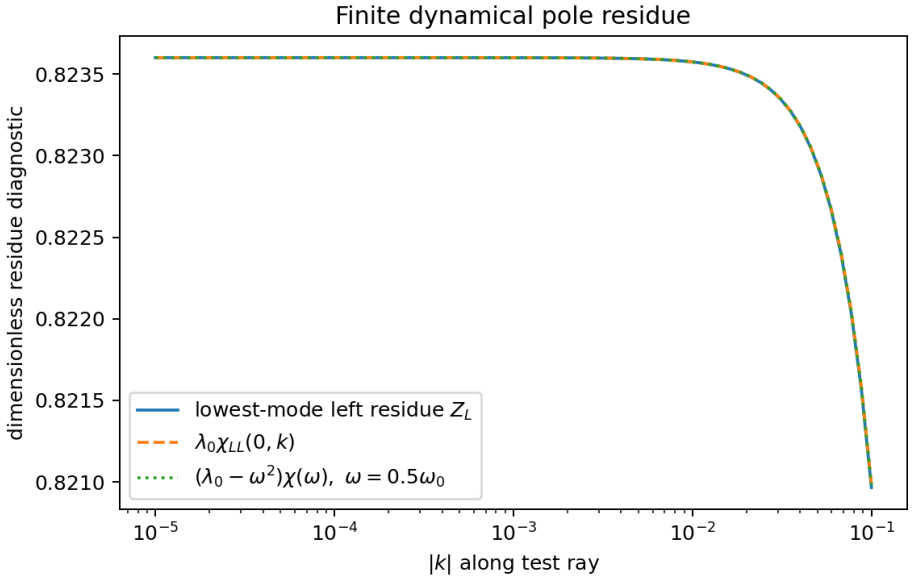

Figure 2: Residue diagnostics. The lowest-mode local residue and the dynamical pole product approach a nonzero plateau.

Ward/projector residual	Maximum
Trace residual $\|\mathrm{tr}\, P_{\mathrm{TT}}\|_\infty$	2.498e-16
Divergence residual $\|n \cdot P_{\mathrm{TT}}\|_\infty$	1.280e-18
Idempotence residual $\|P^2 - P\|_\infty$	4.441e-16
Symmetry residual $\|P - P^T\|_\infty$	0.000e+00

Thus scalar and vector leakage is absent in the projected conserved-source response.

5 Strictly local exact-TT source boundary

A separate source-locality issue remains. The local tensor coordinate has finite TT overlap, but exact polynomial locality plus exact TT conservation is stronger. The known local syzygy starts at Cotton order, with form factor $O(k^3)$. We therefore audit the conservative proxy

$$\chi_{\text{local exact TT}}(0,k) \sim |n(k)|^6 \chi_{LL}(0,k).$$

The fitted exponent is

$$3.999188349441,$$

consistent with k^4 rather than k^{-2}. This does not remove the projected TT pole, but it says that a strictly local self-contained exact-TT source operator is derivative-suppressed.

6 Perturbative Ward stability

First-class closure is kinematic for copy-space stiffnesses because the stiffness is identity on each tensor quotient. The sampled residual for arbitrary positive copy-space blocks is

$$2.581e-15.$$

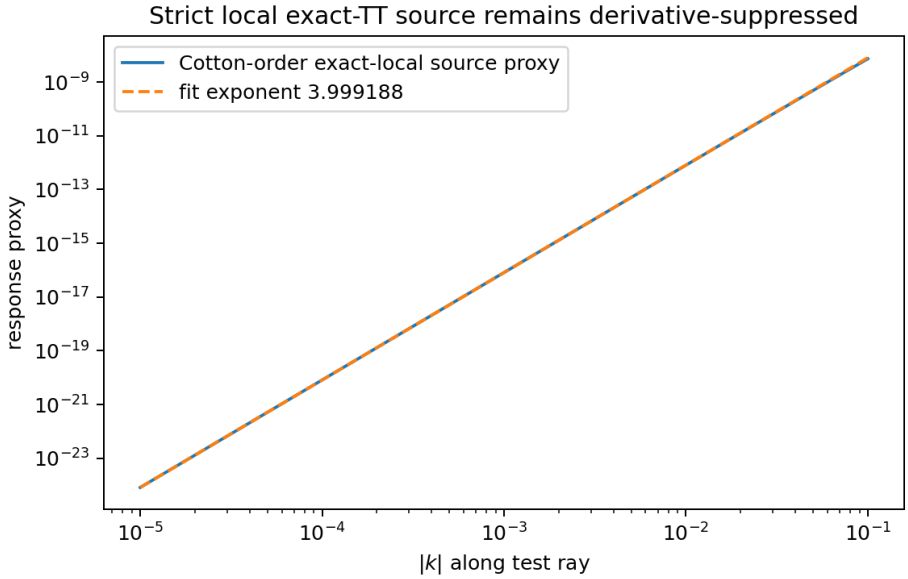

Figure 3: If the source operator itself must be strictly local and exactly TT, the first available module is Cotton-order and the response is suppressed.

Within the mass-forbidden chiral family $r(k) = r_0 + r_1 m_W(k)$, $c(k) = c_1 m_W(k)$, all 48 of 48 random samples preserved one scalar branch, $z \simeq 1$, a finite second gap, and a gapped known doubler. The observed z range was

$$0.999733 \leq z \leq 0.999736.$$

However a constant positive copy-space mass is also local and first class. It gaps the pole at fixed nonzero value. Therefore first-classness by itself does not protect masslessness; a symmetry, tuning condition, or phase mechanism must forbid this relevant perturbation.

7 Verdict

The chiral bosonic lift passes the conserved/projected TT response test: it has a $1/k^2$ static susceptibility, a linear dynamical pole, finite pole residue, and exact transverse-traceless Ward identities. It does not yet pass the stronger phase-stability test because generic first-class local masses are allowed unless an additional symmetry or many-body mechanism forbids them. The next constructive problem is therefore no longer branch count or bosonic positivity; it is protection of the source-coupled TT pole.

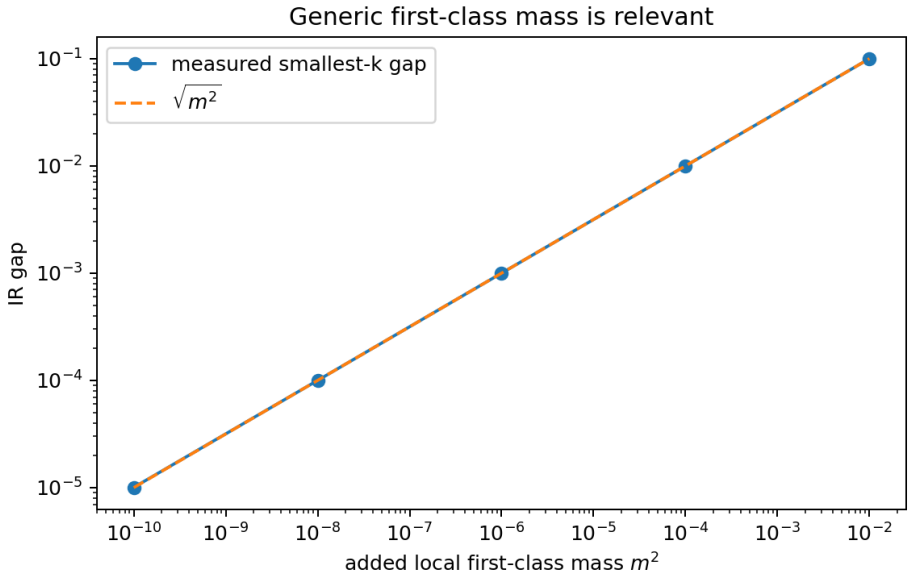

Figure 4: A generic local first-class mass perturbation is relevant and gaps the infrared.

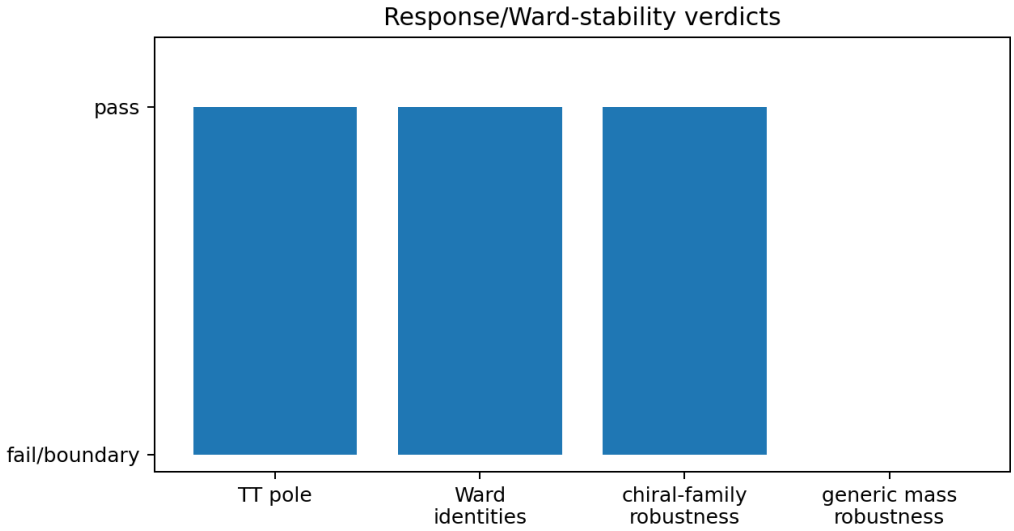

Figure 5: Summary of the response/Ward-stability audit.