CORRECTION AND SCOPE

Microscopic Route to the QATNU Closure Farrow From Coupler Physics to Scalar Geometry: A Microscopic Route to the QATNU Closure

Correction to the opening premise. The QATNU simulator has not established scalar clock closure. Nine source/window repairs rescued zero cases. The medium-specific coupler bridge remains a candidate model.

Manuscript · p. 1, opening premise

A Test Protocol for Scalar Timing Closure and Dimensional Selection

Evidence · Scalar source/window residual audit

From Coupler Physics to Scalar Geometry: A Microscopic Route to the QATNU Closure

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Abstract

The first three papers in the present program establish a scalar observable chain, a timing-first experimental grammar, and a first bound-setting hardware proposal. What remains is the microscopic bridge: why a real medium should generate the screened weighted-Laplacian closure

$$(\kappa L_w + \mu^2 I)\Lambda = A\rho$$

rather than merely serving as a suggestive analogy. We address that bridge for a concrete medium class: reconfigurable superconducting coupled networks with tunable couplers. Starting from a circuit-QED style Hamiltonian, we identify a slow scalar observable—the local dressed-frequency load—and show how, after linearization around an operating point and elimination of fast coupler variables, its leading coarse-grained dynamics take the QATNU form

$$\partial_t \Lambda + \Gamma(\kappa L_w + \mu^2 I)\Lambda = \Gamma A\rho(t)$$

up to nonlinear, higher-order, memory, and thermal corrections. The programmable coupling matrix defines the weighted graph directly, not metaphorically. We derive the associated modal relaxation spectrum, work through a 20-node ring-versus-star example, and connect the resulting transient and static signatures to the timing-first experimental program. The paper does not claim that superconducting arrays are the unique or final source medium. It shows instead that one real, already-existing medium class can plausibly instantiate the operator-level structure assumed by the scalar emergent-geometry program, and thereby turns the first three papers from a coherent proposal set into a medium-specific research platform.

1. Introduction

A speculative theory begins to mature only when its effective equations cease to float above matter and become traceable to some real substrate. The earlier scalar emergent-geometry papers developed three things: a local, measurement-free, graph-based backbone; a scalar observable law for timing and redshift; and an experimental strategy based on timing perturbations before force-level signatures. The unresolved question is the microscopic one: why should any physical medium generate a scalar load field governed, to leading order, by a screened graph Laplacian?

This paper proposes a concrete answer for a specific medium class: superconducting arrays with tunable couplers. This choice is not arbitrary. Such systems already realize programmable weighted graphs with experimentally demonstrated rapid gate or coupling control, low residual static interaction, and stable operation over long windows. IBM reported a tunable-coupling architecture for fixed-frequency transmons with maximum two-qubit gate fidelity of 99.85% and calibration stable over a day, while NIST demonstrated flexible parametric coupling with 30–60 ns gate times and short-timescale fidelities peaking near 99.7%. Rigetti has further demonstrated a modular multichip floating tunable coupler architecture. These are precisely the sorts of systems in which one may ask whether a graph-conditioned scalar load can be sourced, switched, and bounded in the

laboratory.[1] [2] [3]

The central claim is narrow. We do not derive the whole worldview from circuit theory. We derive one thing: a slow scalar observable Λ_i in a tunable coupled network can obey, to leading order,

$$(\kappa L_w + \mu^2 I)\Lambda = A\rho, \tag{1}$$

and dynamically,

$$\partial_t \Lambda + \Gamma(\kappa L_w + \mu^2 I)\Lambda = \Gamma A\rho(t). \tag{2}$$

This is the microscopic route needed to connect the dimensional-selection paper to the timing-first and first-bound experimental papers.

2. Medium class and microscopic Hamiltonian

We consider a network of node modes coupled by tunable elements on graph edges. A minimal microscopic Hamiltonian is

$$H = H_{\text{nodes}} + H_{\text{coup}} + H_{\text{int}} + H_{\text{drive}} + H_{\text{diss}}. \tag{3}$$

The node sector is

$$H_{\text{nodes}} = \sum_i \left[\omega_i a_i^\dagger a_i + \frac{K_i}{2} a_i^\dagger a_i^\dagger a_i a_i \right], \tag{4}$$

where a_i are node-mode annihilation operators, ω_i are bare frequencies, and K_i weak anharmonicities. Coupler modes are written

$$H_{\text{coup}} = \sum_{\langle i,j \rangle} \Omega_{ij} b_{ij}^\dagger b_{ij}, \tag{5}$$

with interactions

$$H_{\text{int}} = \sum_{\langle i,j \rangle} g_{ij}(t)(a_i + a_i^\dagger)(b_{ij} + b_{ij}^\dagger) + g_{ji}(t)(a_j + a_j^\dagger)(b_{ij} + b_{ij}^\dagger). \tag{6}$$

After the usual rotating-frame reduction, bias-point expansion, and elimination of strongly off-resonant sectors, the retained microscopic effective Hamiltonian takes the familiar form

$$H_{\text{eff,micro}} = \sum_i \tilde{\omega}_i n_i + \sum_{\langle i,j \rangle} J_{ij}(t)(a_i^\dagger a_j + a_j^\dagger a_i) + \sum_{\langle i,j \rangle} \chi_{ij} n_i n_j + \cdots, \tag{7}$$

with $n_i = a_i^\dagger a_i$. Equation (7) is not the QATNU closure; it is the microscopic object from which the closure will be extracted.

[1] J. Stehlik *et al.*, "Tunable coupling architecture for fixed-frequency transmon superconducting qubits," *Phys. Rev. Lett.* **127**, 080505 (2021).

[2] X. Jin *et al.*, "Versatile parametric coupling between two statically decoupled transmon qubits," NIST publication (2023); and X. Y. Jin *et al.*, "Fast, tunable high-fidelity CZ gates between superconducting qubits with parametric microwave control of coupling," *Phys. Rev. Applied* accepted (2025).

[3] M. Field *et al.*, "Modular superconducting-qubit architecture with a multichip tunable coupler," *Phys. Rev. Applied* **21**, 054063 (2024).

3. The scalar observable

The first decision is what scalar quantity should play the role of Λ_i. The most useful choice is the *local dressed-frequency load*

$$\Lambda_i \equiv \frac{\delta\omega_i^{\mathrm{dress}}}{\omega_*}, \qquad \delta\omega_i^{\mathrm{dress}} \equiv \tilde{\omega}_i - \omega_i^{(\mathrm{ref})}, \tag{8}$$

normalized to a reference frequency ω_*. This definition is natural for four reasons. First, it is scalar. Second, it is directly tied to timing through phase accumulation and Ramsey-type observables. Third, it is known in superconducting systems that coupler bias, local occupation, and interaction dressing shift effective frequencies. Fourth, it is the scalar quantity most directly matched to the earlier timing-first program.

Other choices are possible—a phase-stiffness load, an effective impedance load, or a local frustration-energy density—but (8) is the cleanest first route because it already speaks the language of timing.

A point requiring immediate clarification is that (8) defines an *internal* scalar load on graph vertices. By itself this does not yet imply an externally measurable field in the space surrounding the chip. There are therefore two experimental regimes. In the *internal-sensor* regime, a spectator qubit, resonator, or Ramsey probe integrated into the same cryogenic graph measures Λ_i directly as a graph-conditioned dressed-frequency load. In the *external-sensor* regime, one posits an exterior coupling map

$$\Lambda_{\mathrm{ext}}(x) = \sum_i C(x,i)\,\Lambda_i, \tag{9}$$

where the kernel $C(x,i)$ must be supplied by an additional physical transmission mechanism, such as electromagnetic near-field leakage, substrate phononic transmission, or some genuinely new scalar channel. The present paper rigorously derives only the internal graph closure. The first-bound and timing-first papers treat (9) as an experimentally testable hypothesis. This distinction matters: an internal null or positive result already constrains the medium mapping, while an external positive result would support the stronger scalar-geometry interpretation.

4. From coupler graph to weighted Laplacian

The programmable coupling graph is encoded in the effective couplings J_{ij} and, more generally, in any local graph-conditioned coefficients extracted from (7). We define a weighted graph by

$$w_{ij} = f(J_{ij}, \chi_{ij}, \bar{n}_i, \bar{n}_j, \text{bias point}), \tag{10}$$

with the leading choice simply

$$w_{ij} \approx |J_{ij}|. \tag{11}$$

This graph is not a visual analogy. It is the physical coupling matrix of the medium itself. The weighted degree matrix is

$$D_{ii} = \sum_j w_{ij}, \tag{12}$$

and the weighted Laplacian is

$$L_w = D - W. \tag{13}$$

At this point the graph-theoretic operator required by the earlier scalar papers has appeared inside the medium, not outside it.

5. Separation of scales and leading closure

The key step is to separate fast and slow variables. Operationally, "eliminate to leading order" means the following. One partitions the microscopic variables into a slow retained sector s (the dressed-frequency loads) and a fast sector f (coupler-conditioned amplitudes, occupations, and bias-relaxation variables), linearizes about a stable operating point, and writes the coupled equations schematically as

$$\dot{s} = A_{ss}s + A_{sf}f + \eta_s, \tag{14}$$

$$\dot{f} = A_{ff}f + A_{fs}s + \eta_f, \tag{15}$$

with $\|A_{ff}\|$ setting a faster timescale than $\|A_{ss}\|$. The leading elimination is then the Markovian adiabatic-elimination step

$$f(t) \approx -A_{ff}^{-1}A_{fs}s(t), \tag{16}$$

which is the zeroth-order truncation of a more general Nakajima–Zwanzig or memory-kernel treatment. Substituting (16) back gives

$$\dot{s} \approx \big(A_{ss} - A_{sf}A_{ff}^{-1}A_{fs}\big)s + \tilde{\eta}_s, \tag{17}$$

so the effective slow generator is the Schur complement of the fast block. Memory corrections enter at the next order when the time dependence of s inside the convolution integral is retained rather than frozen. In the present paper, "leading order" means exactly the Markovian Schur-complement reduction (17); Eq. (26) below then represents the first omitted corrections.

The dressed shift can be expanded as

$$\delta\omega_i^{\mathrm{dress}} = \sum_j c_{ij}X_{ij} + \sum_{jk} c_{ijk}X_{ij}X_{ik} + \cdots, \tag{18}$$

where the X_{ij} are fast coupler-conditioned variables. Linearizing around the reference state sets $X_{ij} = X_{ij}^{(0)} + \delta X_{ij}$ and retains only the first variation,

$$\delta\delta\omega_i^{\mathrm{dress}} \approx \sum_j c_{ij}\,\delta X_{ij}. \tag{19}$$

If neighboring nodes are coupled through shared coupler-conditioned variables, the elimination of δX_{ij} generates a nearest-neighbor redistribution term proportional to differences of the retained scalar load, so that

$$\Lambda_i \approx A\rho_i + \kappa\sum_j w_{ij}(\Lambda_j - \Lambda_i) - \mu^2\Lambda_i, \tag{20}$$

which can be rearranged into the elliptic closure

$$(\kappa L_w + \mu^2 I)\Lambda = A\rho. \tag{21}$$

The parameters have clear meaning: κ is the graph-transport or stiffness coefficient of scalar-load redistribution; μ is a screening scale arising from leakage, damping, finite coherence depth, or environmental relaxation; and $A\rho_i$ is the effective source generated by graph-state switching, local bias imbalance, pumped occupation redistribution, or other departures from the reference configuration.

The point of (20) is not that every medium must yield exactly this form. It is that, under weak driving, near a stable operating point, the leading Markovian elimination of fast variables produces a linear local response operator on the programmable graph. Higher-order nonlinearities, nonlocal kernels, and memory terms are then naturally understood as corrections rather than arbitrary alternatives.

5.1. Why this closure and not another?

Equation (21) is selected as the lowest-order local, linear, isotropic, source-response operator compatible with weak-source superposition, graph relabeling symmetry, and stability of the retained scalar sector. Equivalently, it is the Euler-Lagrange equation of the quadratic functional

$$\mathcal{F}[\Lambda] = \frac{\kappa}{2}\langle \Lambda, L_w \Lambda\rangle + \frac{\mu^2}{2}\|\Lambda\|^2 - A\langle \rho, \Lambda\rangle. \tag{22}$$

The first term penalizes roughness across strongly coupled edges, the second unsustained bulk load, and the third injects source structure. Nonlinear or higher-order closures,

$$(\kappa L_w + \mu^2 I)\Lambda + \beta\Lambda^2 = A\rho, \qquad (\kappa_1 L_w + \kappa_2 L_w^2 + \mu^2 I)\Lambda = A\rho,$$

are not excluded; they are simply next-order terms in the effective theory and should be treated as corrections unless the leading linear closure fails.

5.2. Weight choice and robustness

Equation (11) takes $w_{ij} \approx |J_{ij}|$ as the leading choice, but the more complete definition includes χ_{ij}, operating occupations, and the bias point. In practice this means the spectrum of L_w is perturbed by weight disorder,

$$w_{ij} = w_{ij}^{(0)} + \delta w_{ij}. \tag{23}$$

For the ring-versus-star example below, the qualitative relaxation ordering is robust so long as the disorder scale satisfies $|\delta w_{ij}| \ll w$ and does not destroy the gross graph class itself. Standard matrix perturbation theory gives eigenvalue shifts bounded by the operator norm of the Laplacian perturbation,

$$|\delta\lambda_n| \le \|\delta L_w\|, \tag{24}$$

so the nearly order-of-magnitude separation between the slowest ring and star modes survives moderate nonuniformity. What will not be robust are precise quantitative estimates of τ_n in heavily disordered devices; those require the full weight map rather than the leading uniform approximation.

A fully explicit algebraic derivation for a two-node and three-node toy model is given in Appendix A. Its purpose is not realism but transparency: it shows step by step how the Markovian Schur-complement reduction generates a screened Laplacian-like closure and where the first memory correction appears.

6. Dynamical completion

For switching and transient measurements one needs time dependence. The natural dynamical completion is

$$\partial_t \Lambda + \Gamma(\kappa L_w + \mu^2 I)\Lambda = \Gamma A \rho(t), \tag{25}$$

with Γ setting the overall relaxation rate. Diagonalizing

$$L_w \phi_n = \lambda_n \phi_n, \tag{26}$$

we obtain the mode timescales

$$\tau_n^{-1} = \Gamma(\kappa \lambda_n + \mu^2). \tag{27}$$

This equation is crucial because it generates a concrete transient discriminant: different graph topologies produce different relaxation spectra even when total chip mass is essentially fixed.

7. Worked example: 20-node ring versus 20-node star

A specific graph pair clarifies the formalism. Consider a 20-node ring with uniform nearest-neighbor weight w. Its Laplacian eigenvalues are

$$\lambda_k^{\text{ring}} = 2w\left(1 - \cos\frac{2\pi k}{20}\right), \qquad k = 0, \ldots, 19. \tag{28}$$

The slowest nonzero mode is

$$\lambda_1^{\text{ring}} = 2w\left(1 - \cos\frac{2\pi}{20}\right) \approx 0.0979\, w. \tag{29}$$

Now consider a 20-node star with one hub and 19 leaves, also with edge weight w. Its Laplacian spectrum is

$$\{0,\ w \text{ (mult. 18)},\ 20w\}. \tag{30}$$

The slowest nonzero mode is therefore

$$\lambda_1^{\text{star}} = w. \tag{31}$$

Hence the slowest relaxation scales are

$$\tau_{\text{ring},1}^{-1} = \Gamma(\kappa \lambda_1^{\text{ring}} + \mu^2), \qquad \tau_{\text{star},1}^{-1} = \Gamma(\kappa w + \mu^2). \tag{32}$$

In the weak-screening regime $\mu^2 \ll \kappa w$, the star relaxes roughly ten times faster than the ring. This difference is not due to mass; it is due to graph-conditioned scalar transport. For the static channel the response also differs because

$$\Lambda = A(\kappa L_w + \mu^2 I)^{-1} \rho \tag{33}$$

depends on the full inverse operator, not merely the total edge weight.

8. Screening, thermal channels, and correction hierarchy

The closure only matters if screening does not immediately kill the signal. In a tabletop experiment one must treat μ as a primary unknown. If $\mu r \gtrsim 1$ over the measurement baseline, the response becomes Yukawa-suppressed and the static signal can vanish. Bounding or measuring μ through distance-law discrimination is therefore an early objective, not an afterthought.

Equally important is the distinction between scalar relaxation and ordinary cryogenic transients. Coupler switching changes microwave field distributions and can deposit small amounts of heat. Dilution refrigerators also support mechanical and thermal relaxation channels with their own characteristic timescales. The scalar prediction from (27), however, is graph-spectral: changing ring to star changes the modal set $\{\lambda_n\}$, and therefore the transient response in a topology-dependent way. Thermal tails, by contrast, should track cryostat heat capacities, conductances, and switching dissipation more directly than graph Laplacian spectra. This difference does not guarantee clean separation in practice, but it provides a nontrivial discriminant.

The leading closure is expected to break when any of the following become important:

$$(\kappa L_w + \mu^2 I)\Lambda = A\rho + \beta\Lambda^2 + \eta L_w^2 \Lambda + \int_0^t M(t-s)\Lambda(s)\, ds + \cdots . \tag{34}$$

These corrections encode nonlinearity, higher-order graph structure, memory kernels, anisotropy, and thermal backaction. Their presence would not invalidate the program; it would delimit the regime in which the linear scalar closure is trustworthy.

9. Connection to the first-bound experiment

The present paper supplies the microscopic content missing from the first-bound proposal. A timing-first observable is built from the same scalar load field defined in (8). For an internal-sensor experiment, the measured signal is simply a function of the vertex loads themselves. For an exterior-sensor experiment, one must additionally specify the map (9) from internal loads to the sensor position. The microscopic derivation in this paper therefore guarantees the internal graph-conditioned source dynamics; the stronger claim of an external scalar geometry requires, in addition, a physically realized coupling kernel $C(x, i)$. A topology-switch experiment at approximately fixed mass is then sensitive to

$$\Delta\Lambda_{\text{topo}} = A\left[(\kappa L_w^{(1)} + \mu^2 I)^{-1}\rho^{(1)} - (\kappa L_w^{(2)} + \mu^2 I)^{-1}\rho^{(2)}\right]. \tag{35}$$

The transient channel is governed by the corresponding Laplacian spectra and (27). In this way the medium paper, the timing-first experimental paper, and the first-bound superconducting array proposal become one connected arc rather than three adjacent ideas.

10. Discussion

The point of the paper is not to show that superconducting arrays are the unique source medium of scalar emergent geometry. It is to show that the weighted-Laplacian closure used in the earlier

papers can arise as the leading effective equation for a physically identifiable slow scalar observable in a real medium class. This is already enough to change the status of the program. One can now ask medium-specific questions: how do κ, μ, and Γ scale with coupler bias? What graph families maximize topology change at fixed mass and fixed dissipation? What switching protocols distinguish graph-spectral relaxation from ordinary thermal tails? Which media besides superconducting arrays admit the same coarse-grained structure?

The hardest next step remains open. One would like either a medium-class theorem that derives (1) from broad conditions, or a sharper microscopic analysis that computes κ, μ, Γ, and ρ from a fully specified experimental device. But the present result is already nontrivial: it bridges the earlier abstract operator closure to a medium that can be built, switched, and measured.

11. Conclusion

A physical theory is strengthened when what once appeared as a postulate can be seen as the first term of an effective derivation. In reconfigurable superconducting coupled networks, the programmable coupling graph defines a weighted Laplacian directly. A local dressed-frequency load provides a natural scalar observable tied to timing. Under a reasonable separation of scales, the elimination of fast coupler variables yields a leading screened weighted-Laplacian closure for that scalar field, together with graph-spectral transient predictions. This does not complete the scalar emergent-geometry program. It does something more important at the present stage: it gives the program a plausible microscopic footing in one real medium class.

A. Two-node elimination example

To make the elimination step explicit, consider two retained node loads collected in $s = (\Lambda_1, \Lambda_2)^T$ and one fast coupler variable $f = X$. Linearizing near an operating point gives

$$\dot{\Lambda}_1 = -a\Lambda_1 + bX + A\rho_1, \tag{36}$$

$$\dot{\Lambda}_2 = -a\Lambda_2 + bX + A\rho_2, \tag{37}$$

$$\dot{X} = -\Omega X + c(\Lambda_1 + \Lambda_2) + d(\Lambda_2 - \Lambda_1), \tag{38}$$

with $\Omega \gg a$. The adiabatic leading-order elimination sets

$$X \approx \frac{c(\Lambda_1 + \Lambda_2) + d(\Lambda_2 - \Lambda_1)}{\Omega}. \tag{39}$$

Substituting back and retaining the antisymmetric component yields

$$\dot{\Lambda}_1 \approx -\Big(a + \frac{bd}{\Omega}\Big)\Lambda_1 + \frac{bd}{\Omega}\Lambda_2 + A\rho_1 + \text{symmetric terms}, \tag{40}$$

$$\dot{\Lambda}_2 \approx -\Big(a + \frac{bd}{\Omega}\Big)\Lambda_2 + \frac{bd}{\Omega}\Lambda_1 + A\rho_2 + \text{symmetric terms}. \tag{41}$$

Writing this in vector form and absorbing symmetric renormalizations into μ^2 gives

$$\partial_t \Lambda + \Gamma(\kappa L + \mu^2 I)\Lambda = \Gamma A \rho, \tag{42}$$

with L the two-node graph Laplacian. If one does not freeze X instantaneously, the elimination produces a retarded convolution kernel, which is the origin of the memory correction discussed in the main text.