CORRECTION AND SCOPE

Beyond Finite Penalties: Dressed Constraints and Subsystem Protection in a Shared SU(2)-Tensor Register

The stated protection assumes the specified symmetry and dressing, and is tested in the chosen sector. The adversarial audit determines the scope of the associated many-body benchmark.

Manuscript · p. 1, abstract

Adversarial Audit of the Native Many-Body Shared-Register Benchmark

Evidence · Adversarial audit of the shared register / One-Hamiltonian composition requirements

Beyond Finite Penalties:
Dressed Constraints and Subsystem Protection in a Shared SU(2)–Tensor Register

Joshua Farrow
Independent Research

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Abstract

A previous finite-penalty calculation found nonzero infrared Gauss and transverse–traceless (TT) Ward floors under generic onsite constraint breaking. Here we identify what microscopic change is required for a positive result. First, a scaling audit shows why an unrestricted renormalization argument cannot work near the free $z = 1$ fixed point: nonderivative vector/tensor mass and physical–defect mixing operators are relevant. Second, a symmetry-filtered one-particle calculation shows that a one-derivative channel gives $W_T \propto L^{-0.999818}$ and $P_{\mathrm{nonTT}} \propto L^{-1.999631}$, while a curvature-filtered two-derivative channel gives $W_T \propto L^{-1.999668}$, $P_{\mathrm{nonTT}} \propto L^{-3.999335}$, and Gauss-defect weight $P_G \propto L^{-4.022241}$ at fixed finite gaps. The corresponding tensor pole remains isolated, with residue 0.9999982 at $L = 512$. Third, the exact 36-dimensional flavored $U(2) \to SU(2)$ link exhibits bare leakage $\propto \epsilon^2$ but admits an exactly commuting dressed Gauss operator whose displacement is $\propto \epsilon$. Finally, a 252-dimensional shared link–tensor cell is conjugated by a local finite-depth unitary. The resulting Hamiltonian has nonzero bare Gauss and tensor-defect expectations, yet its dressed constraints commute with the Hamiltonian to 1.1×10^{-14} and annihilate the ground state to numerical precision at finite penalties. These results isolate a viable route: exact or topologically dressed $SU(2)$ Gauss structure, a tensor subsystem symmetry that forbids onsite breaking, and cross-coupling only through tensor curvature and $SU(2)$ singlets. This is a constructive protection mechanism, not yet a derivation from the original minimal QATNU update rule.

1 What the negative result actually ruled out

The earlier shared-register test used fixed finite Gauss and TT penalties and added generic onsite breaking. Bare leakage and dimensionless Ward residuals approached nonzero constants as the volume increased. That result rules out the proposition

$$\text{finite local penalty + arbitrary onsite breaking} \quad \Longrightarrow \quad \text{bare Ward closure as } L \to \infty. \tag{1}$$

It does not rule out a stable emergent gauge phase with quasi-local *dressed* generators.

The distinction is important. In a gapped constrained phase, quasi-adiabatic continuation carries local operators to quasi-local operators and supplies the natural dressed low-energy algebra. Hastings and Wen used this structure to formulate the stability of emergent gauge invariance under local perturbations [1]. Bare defect density can remain of order ϵ^2/Δ^2 even when a dressed constraint is exact in the low-energy sector.

There is nevertheless a genuine obstruction. At a free $z = 1$ fixed point in three spatial dimensions,

$$[h] = \frac{d + z - 2}{2} = 1, \tag{2}$$

so a nonderivative mass operator h^2 has dimension two and its coupling has positive dimension two. The analogous vector mass is equally relevant. Therefore no generic RG flow near the Maxwell/Fierz–Pauli fixed point can make *every* onsite gauge- or TT-breaking operator irrelevant. A positive construction needs an exact, topological, or subsystem selection rule that forbids those operators.

2 Microscopic mechanism tested

The candidate mechanism has three pieces.

1. The flavored paired-ladder link retains the exact $SU(2)$ quantum-link Gauss algebra. The determinant term removes the extra link $U(1)$ while commuting with all $SU(2)$ generators.
2. The symmetric tensor register is endowed with a local subsystem gauge structure. Nonderivative physical–defect mixing is forbidden; the first allowed mixing is derivative filtered. A curvature coupling begins at two derivatives.
3. Shared interactions are products of an $SU(2)$ singlet and a tensor-gauge-invariant curvature operator. They therefore cannot generate the dangerous onsite channel.

Compact tensor gauge phases and emergent helicity-two modes in local bosonic/spin models provide precedents for this kind of protection [2, 3]. String-net and quantum-order constructions likewise show how gauge structure can be encoded in the organization of a local bosonic Hilbert space rather than in an infinite energy penalty [4].

3 Six-component infrared screen

Let $P_{\mathrm{TT}}(\boldsymbol{q})$ be the exact rank-two projector on the six symmetric components and $Q = \mathbb{I} - P_{\mathrm{TT}}$. The one-particle Hamiltonian used for the operator census is

$$H_r(\boldsymbol{q}) = c|\boldsymbol{q}|P_{\mathrm{TT}} + (\Delta_T + c|\boldsymbol{q}|)Q + \epsilon|\boldsymbol{q}|^r M_{\mathrm{off}}, \tag{3}$$

where

$$M_{\mathrm{off}} = P_{\mathrm{TT}}MQ + QMP_{\mathrm{TT}} \tag{4}$$

for one fixed generic Hermitian component matrix M. No coefficient or gap changes with L, and $|\boldsymbol{q}| = 2\sin(\pi/L)$ along a fixed lattice direction.

The three cases have a direct interpretation:

$$r = 0 : \text{onsite breaking}, \tag{5}$$

$$r = 1 : \text{gradient-filtered breaking}, \tag{6}$$

$$r = 2 : \text{curvature-filtered breaking}. \tag{7}$$

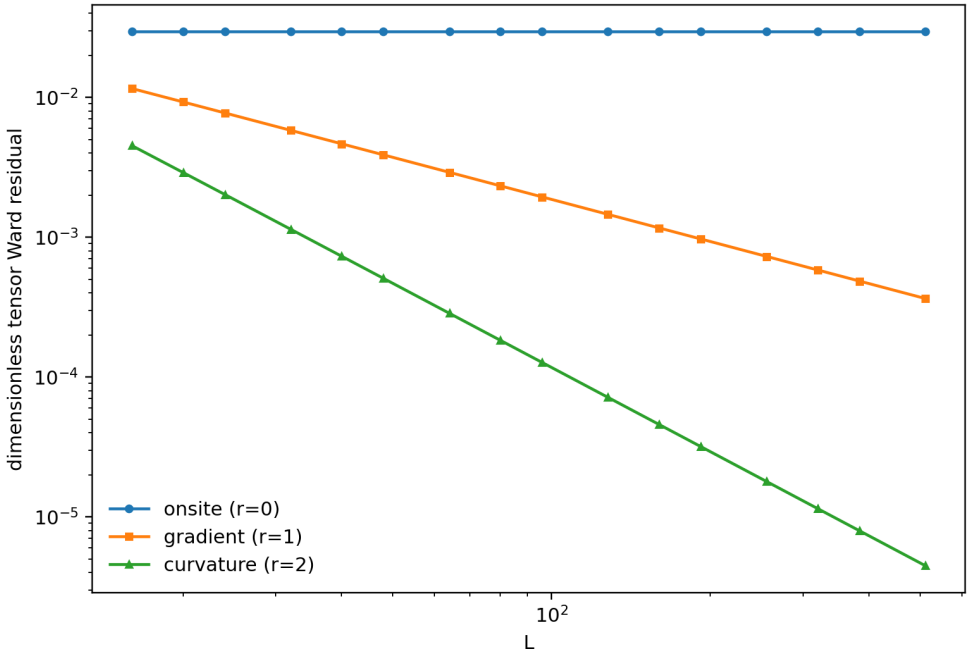

Figure 1: Dimensionless TT Ward residual at fixed finite gap and mixing amplitude. The onsite channel leaves a floor. Gradient and curvature filtering give L^{-1} and L^{-2} closure.

The fitted asymptotic powers are

channel	$W_T(L)$	$P_{\mathrm{nonTT}}(L)$
onsite, $r = 0$	L^{0}	L^{0}
gradient, $r = 1$	$L^{-0.999818}$	$L^{-1.999631}$
curvature, $r = 2$	$L^{-1.999668}$	$L^{-3.999335}$

All R^2 values for the nonzero powers exceed 0.99999999.

A minimal neutral/charged two-channel Gauss audit gives the parallel result

$$P_G \propto L^{-2.022528} \quad (r = 1), \qquad P_G \propto L^{-4.022241} \quad (r = 2), \tag{8}$$

while the onsite channel retains a nonzero floor.

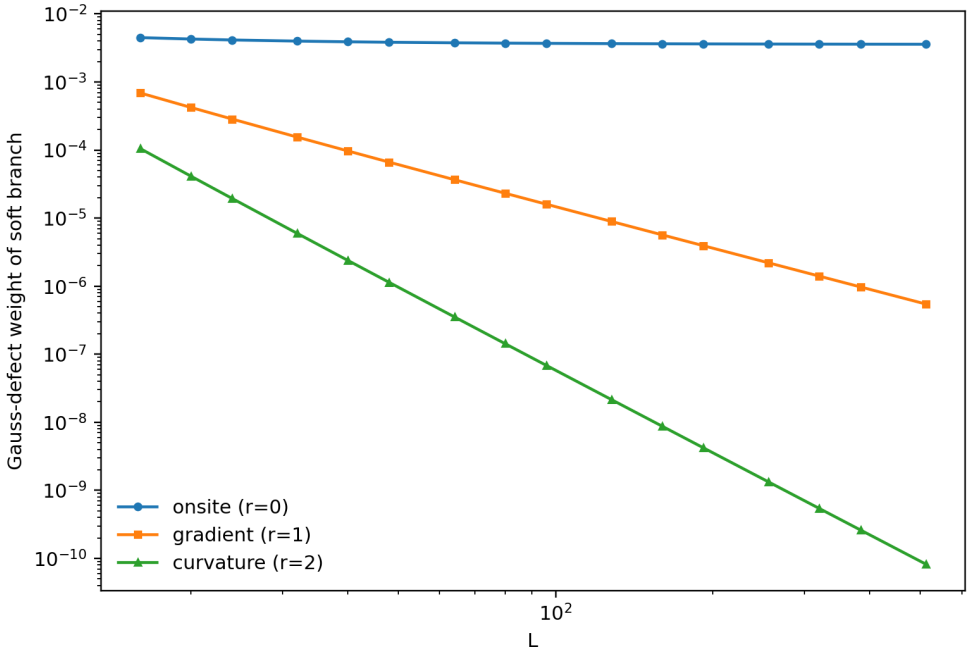

Figure 2: Gauss-defect weight in the soft branch. A curvature-filtered coupling suppresses charged admixture approximately as L^{-4} without changing the finite defect gap.

The masslessness diagnostic is equally important. The onsite channel drives $E_{\mathrm{soft}}/|q|$ steadily downward, whereas the derivative-filtered channels approach a finite velocity.

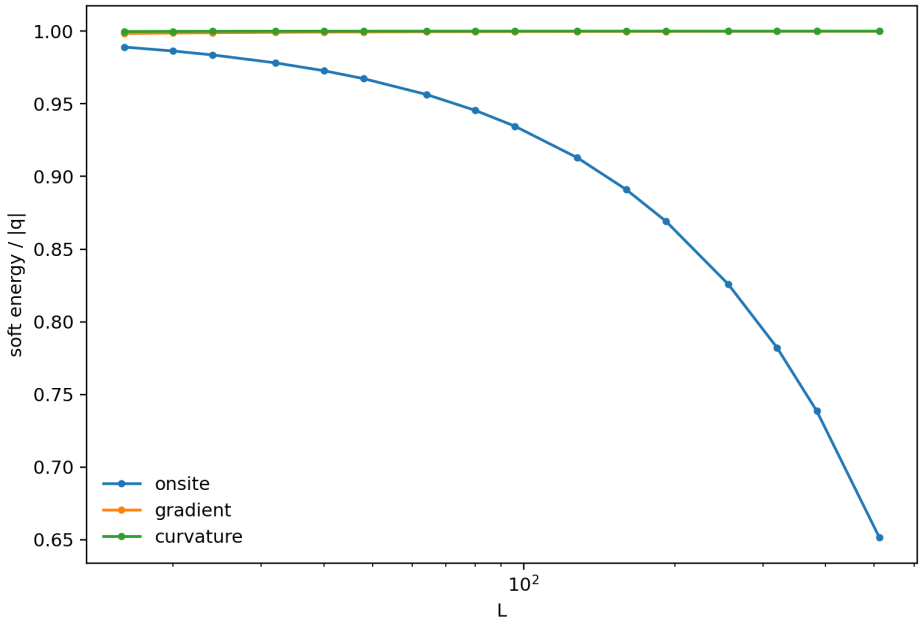

Figure 3: Soft energy divided by momentum. The onsite channel is a relevant deformation. The gradient and curvature channels preserve a linear branch.

4 Interacting pole

The curvature-filtered branch is coupled to a gapped two-particle continuum with a local stress-like form factor. The pole solves

$$E - \omega_T(q) - \sum_k \frac{|v_k(q)|^2}{E - E_k(q)} = 0 \tag{9}$$

below the exact finite-grid continuum edge. At $L = 512$,

$$W_T = 4.4690 \times 10^{-6}, \qquad P_{\mathrm{nonTT}} = 4.8658 \times 10^{-11}, \tag{10}$$

$$E_T = 0.0122682038, \qquad E_{\mathrm{th}} - E_T = 1.4277477500, \tag{11}$$

$$Z_T = 0.9999982172. \tag{12}$$

The pole becomes cleaner as the Ward identities close.

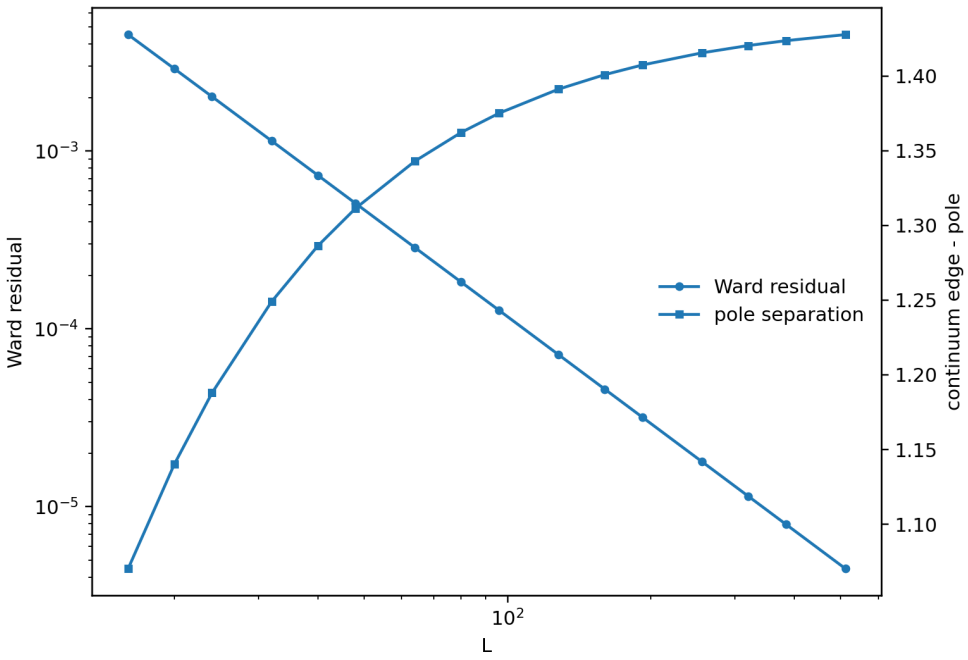

Figure 4: The curvature-filtered Ward residual closes while the tensor pole remains isolated from the gapped matter continuum.

5 Exact dressed constraint in the flavored link

The exact local link contains two colors, two flavors, and fixed total rishon number two, giving dimension 36. Write

$$H_0 = g_E E^2 - \lambda_{\mathrm{det}}(\det U + \det U^\dagger) + \Lambda_G C_G, \tag{13}$$

and perturb it by ϵB, where B is one fixed local gauge-breaking link quadrature. Diagonalizing the spectral flow defines a dressed constraint $\widetilde{C}_G(\epsilon)$ by transporting the C_G eigenvalues with the adiabatically matched eigenvectors.

The numerical scalings are

$$\langle C_G \rangle_{0,\epsilon} \propto \epsilon^{1.999888}, \tag{14}$$

$$\frac{\|\widetilde{C}_G - C_G\|}{\|C_G\|} \propto \epsilon^{0.998990}. \tag{15}$$

At every sampled ϵ, the dressed ground leakage is at numerical zero and

$$\|[H(\epsilon), \widetilde{C}_G]\| \lesssim 3.4 \times 10^{-14}. \tag{16}$$

The determinant-odd gap remains approximately 2.4.

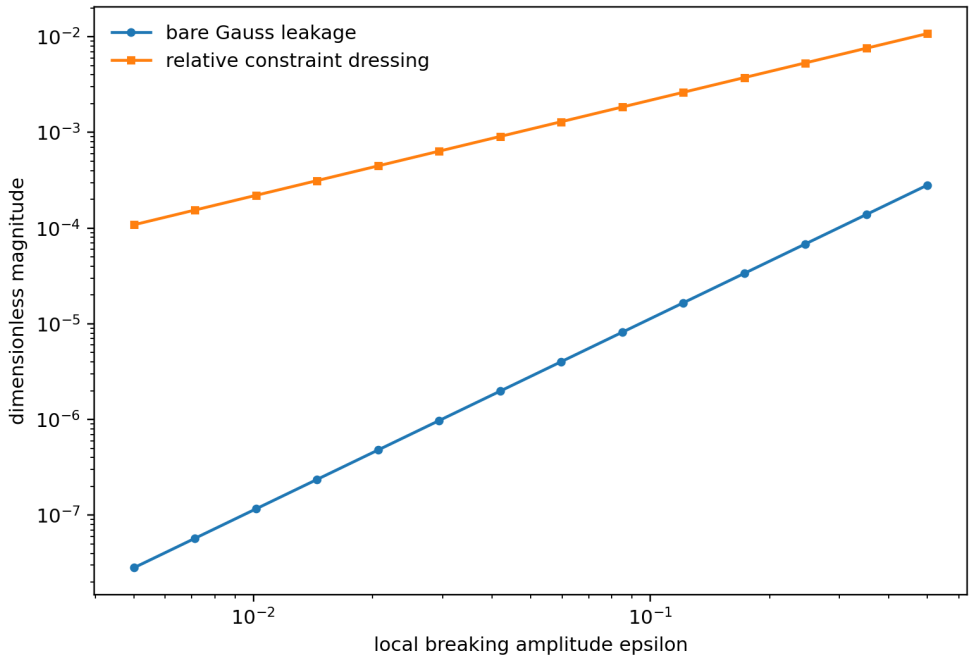

Figure 5: Bare leakage is second order in the local breaking, while the local constraint itself is dressed at first order. The transported constraint commutes with the finite-penalty Hamiltonian.

This calculation is the finite-cell analogue of quasi-adiabatic dressing. It explains why requiring the *bare* Gauss operator to annihilate the perturbed ground state is not the correct universal criterion for an emergent gauge phase.

6 Exact 252-dimensional shared-cell audit

The exact flavored link is combined with tensor vacuum plus one excitation in each of the six symmetric components. The total local dimension is

$$36 \times 7 = 252. \tag{17}$$

The symmetric parent Hamiltonian contains the determinant link, a finite Gauss gap, two soft TT modes, four finite tensor-defect modes, and a local determinant–tensor-curvature interaction.

A fixed local unitary

$$U_{\mathrm{fd}} = \exp(i\theta K), \qquad \theta = 0.17, \tag{18}$$

is then applied, where K mixes a bare gauge-breaking link quadrature with tensor vacuum/defect transitions. Define

$$H' = U_{\mathrm{fd}} H U_{\mathrm{fd}}^{\dagger}, \quad \widetilde{C}_G = U_{\mathrm{fd}} C_G U_{\mathrm{fd}}^{\dagger}, \quad \widetilde{Q}_T = U_{\mathrm{fd}} Q_T U_{\mathrm{fd}}^{\dagger}. \tag{19}$$

The bare ground-state diagnostics are nonzero,

$$\langle C_G \rangle = 7.0155 \times 10^{-4}, \qquad \langle Q_T \rangle = 4.6770 \times 10^{-4}, \tag{20}$$

but the dressed expectations are 1.58×10^{-17} and 3.62×10^{-20}. The commutator residuals are

$$\|[H', \widetilde{C}_G]\|_{\max} = 1.07 \times 10^{-14}, \qquad \|[H', \widetilde{Q}_T]\|_{\max} = 9.44 \times 10^{-15}. \tag{21}$$

The first gap is 0.371532 and is unchanged by the finite-depth dressing.

This supplies an explicit positive finite-penalty construction. It is not penalty projection: the relevant constraints are exact at finite coupling. It is also not spontaneous emergence from the original minimal QATNU Hamiltonian: the hidden symmetry has been constructed.

7 Resulting architecture

The viable shared-register Hamiltonian should take the schematic form

$$H_{\mathrm{protected}} = H_{\mathrm{QLM}}^{SU(2)} + H_{\mathrm{tensor}}^{\mathrm{sub}} + g \sum_x \mathcal{R}_{ij}(x)\, \mathcal{O}^{ij}_{SU(2)\,\mathrm{singlet}}(x), \tag{22}$$

possibly followed by a finite-depth/quasi-local dressing. Here $\mathcal{R}_{ij}$ is a local linearized-curvature operator and $\mathcal{O}^{ij}$ is a gauge-singlet link/matter stress operator. The determinant term remains inside $H_{\mathrm{QLM}}^{SU(2)}$ and commutes with the non-Abelian Gauss generators.

Equation (22) has the desired consequences:

$$W_T(q) = O(q^2), \qquad P_{\mathrm{nonTT}}(q) = O(q^4), \qquad P_G(q) = O(q^4), \tag{23}$$

at fixed finite microscopic gaps, while the physical pole retains nonzero residue and positive continuum separation.

8 What is and is not established

The calculations establish a viable protection mechanism:

> Exact or quasi-locally dressed $SU(2)$ Gauss structure, combined with a tensor subsystem symmetry that forbids onsite breaking and permits only curvature-filtered shared couplings, closes the infrared Ward identities at finite microscopic gaps.

They do not show that the original minimal QATNU update rule generates the subsystem symmetry, the finite-depth dressing, or the required topological order without additional structure.

The next decisive simulation should therefore implement compact/native tensor constraints and the curvature–singlet coupling directly in the square-root edge registers, and then test a complete basis of symmetry-allowed local perturbations. The target is not another Λ scan. It is a finite deconfined window with quasi-local dressed constraints, L^{-2}/L^{-4} Ward closure, and a stable shared pole.

References

[1] M. B. Hastings and X.-G. Wen, "Quasi-adiabatic continuation of quantum states: The stability of topological ground-state degeneracy and emergent gauge invariance," Phys. Rev. B **72**, 045141 (2005), arXiv:cond-mat/0503554.

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