CORRECTION AND SCOPE

Small Hard-Quotient Scalar Chain: Exact Diagonalization Pilot

Original title · Microscopic Finite-Spin ED Pilot for the Hard-Constrained Ward-Square Tensor Model

Exact diagonalization of a small hard-quotient scalar chain. The finite Hilbert space, imposed quotient and single scalar chain delimit the result.

Manuscript · p. 1, pilot verdict

Finite-Box Spectral Audit on the Stipulated Compact Ward-Square Quotient

Evidence · Reduced finite-spin tower scaling

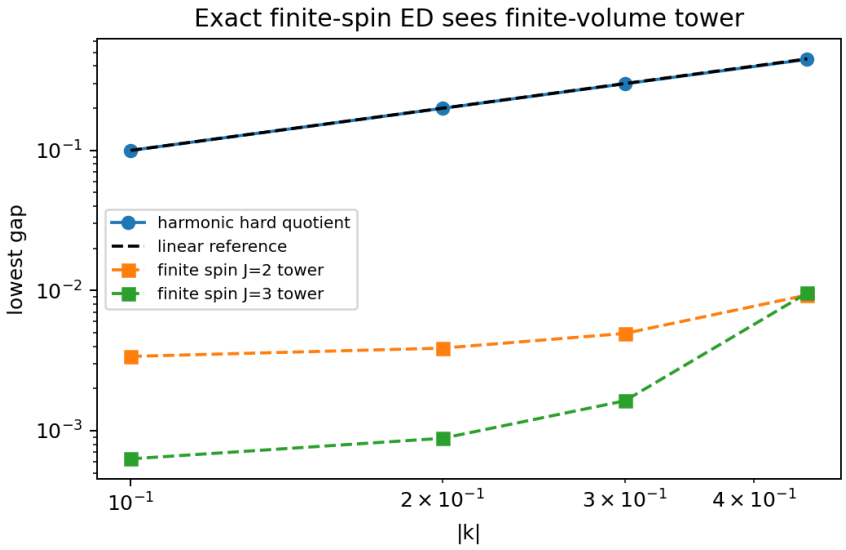
Microscopic Finite-Spin ED Pilot for the Hard-Constrained Ward-Square Tensor Model

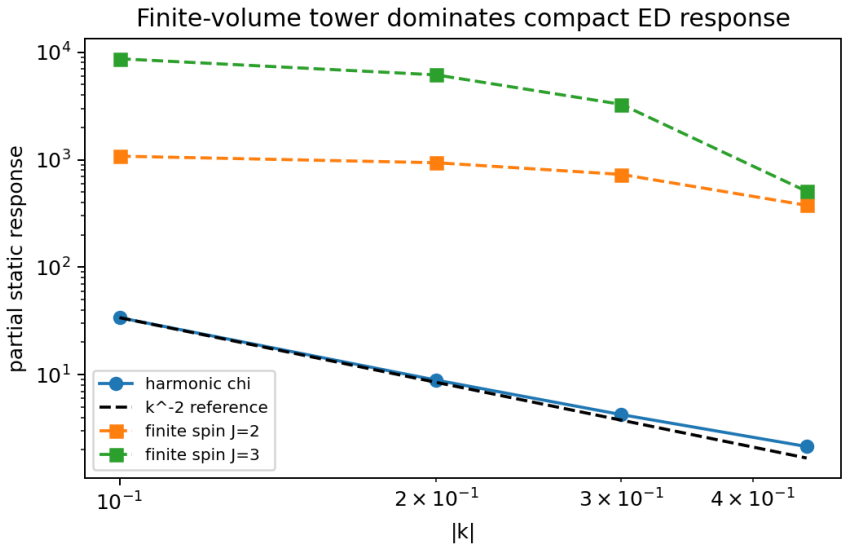
Verdict

An explicit finite-spin operator Hamiltonian was constructed and diagonalized on the hard-quotient scalar chain. The calculation is useful, but it is not yet a final nonperturbative finite-spin phase proof. On tiny exact-diagonalization volumes the compact diagonal U(1) produces an Anderson tower, and the local wall source couples strongly to that tower. The thermodynamic ordered spin-wave pole is therefore not cleanly isolated by this small ED alone.

Hard-constrained harmonic target

The matching harmonic quotient for W=2 has z=0.994961574, chi exponent=-1.842630408, and mean lambda chi residue=0.374850930; the expected residue is 1/(W+1)=0.333333333.





Exact finite-spin operator construction

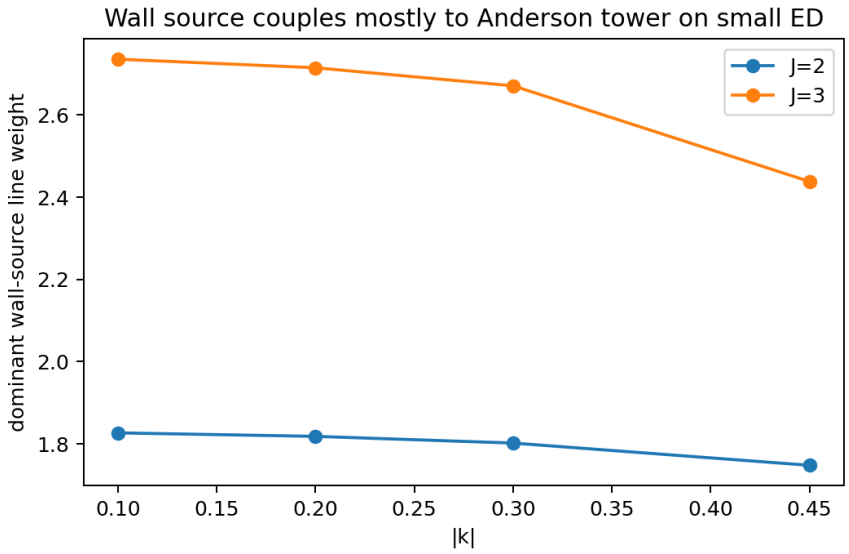
Each hard-quotient coordinate is represented by a spin-J degree of freedom using q=S_x/sqrt(J) and p=S_y/sqrt(J). The Hamiltonian is H=1/2 p^2 + 1/2 q K q with local path stiffness and spatial stiffness. This is a finite-dimensional microscopic operator model for the quotient chain.

J=2 exact diagonalization; local dimension 5; Hilbert dimension 125

k	tower gap	partial chi	dominant line	dominant gap	dominant weight
0.100	3.387e-03	1.079e+03	1	3.387e-03	1.827e+00
0.200	3.881e-03	9.370e+02	1	3.881e-03	1.818e+00
0.300	4.941e-03	7.295e+02	1	4.941e-03	1.802e+00
0.450	9.292e-03	3.762e+02	1	9.292e-03	1.748e+00

J=3 exact diagonalization; local dimension 7; Hilbert dimension 343

k	tower gap	partial chi	dominant line	dominant gap	dominant weight
0.100	6.292e-04	8.696e+03	1	6.292e-04	2.736e+00
0.200	8.821e-04	6.156e+03	1	8.821e-04	2.715e+00
0.300	1.632e-03	3.273e+03	1	1.632e-03	2.671e+00
0.450	9.603e-03	5.078e+02	1	9.603e-03	2.438e+00



Local constraint generation check

L	W	rank	expected	null dim	expected	matches
2	1	58	58	38	38	True
2	2	87	87	57	57	True
3	1	210	210	114	114	True

Boundary and next step

The pilot validates an actual finite-spin operator realization of the hard-quotient chain and confirms the local integer constraint ranks. It also shows why direct tiny-volume ED is not the clean final observable: the Anderson tower must be separated from the spin-wave pole. The next decisive calculation is a transfer-matrix/QMC dynamic-structure-factor audit, or a larger hard-constraint finite-spin simulation with tower filtering.