Local Matter/Ward Coupling Audit for the Chiral Bosonic Spin-2 Carrier

QATNU numerical audit

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Abstract

We test the next source-coupling question for the chiral continuous-variable spin-2 carrier: can the transverse-traceless (TT) source response be supplied by a local Ward/conservation sector, rather than by inserting a nonlocal TT projector by hand? The answer is positive for an auxiliary local stress sector with a local conservation constraint $d_i S_{ij} = 0$: eliminating the constraint gives an $O(1)$ TT overlap and the same $1/k^2$ pole seen by externally conserved TT sources. The audited Ward-constrained response exponent is -2.000001654952, and a random constrained stress has mean TT weight fraction 0.663459, consistent with the 2/3 dimension ratio inside the divergence-free subspace. The boundary is also explicit: derivative-built strictly local conserved source operators remain suppressed. Beltrami double-curl sources give a k^2 response, and exact local Cotton-order TT sources give a k^4 response. Thus the missing ingredient can be a local Ward sector, but a full microscopic matter Hamiltonian or universal nonlinear coupling is not yet constructed.

1 Question tested

The current chiral carrier already has a positive scalar boundary response

$$\chi_0(0,k) \sim \frac{1}{k^2} \tag{1}$$

and an exact two-dimensional TT quotient. Projected or externally conserved finite-amplitude TT sources therefore see the intended spin-2 pole. The remaining question is sharper: can a *local* source sector provide the Ward projection dynamically?

We compare three source classes:

1. an externally conserved or projected TT source;
2. an auxiliary local symmetric stress variable S_{ij} subject to a local conservation constraint $d_i S_{ij} = 0$;
3. strictly local derivative-built source operators, such as Beltrami double-curl and Cotton sources, whose conservation or exact TT property holds as a polynomial identity.

2 Local Ward-constrained stress sector

For each nonzero momentum, let $D(k)$ be the 3×6 divergence matrix mapping a symmetric tensor S_{ij} to $d_i S_{ij}$. The local Ward constraint is

$$D(k)S(k) = 0. \tag{2}$$

The null space of this constraint is three-dimensional: two TT components plus one transverse trace-like component. Projecting a local symmetric stress variable into this constraint surface is a local

constrained-source construction; it is not the same as using a nonlocal TT source operator as a microscopic observable.

The TT projector P_{TT} lies inside the divergence-free projector P_{div}. Numerically,

$$\|DP_{\mathrm{div}}\|_\infty \le 1.224 \times 10^{-17}, \tag{3}$$
$$\|DP_{\mathrm{TT}}\|_\infty \le 7.274 \times 10^{-17}, \tag{4}$$
$$\|\mathrm{tr}\, P_{\mathrm{TT}}\|_\infty \le 1.665 \times 10^{-16}, \tag{5}$$
$$\|P_{\mathrm{TT}}^2 - P_{\mathrm{TT}}\|_\infty \le 3.331 \times 10^{-16}. \tag{6}$$

A random stress projected into the divergence-free Ward subspace has mean TT weight fraction

$$0.663459, \tag{7}$$

consistent with the expected 2/3 dimension ratio. Thus the constrained local stress sector has $O(1)$ TT content.

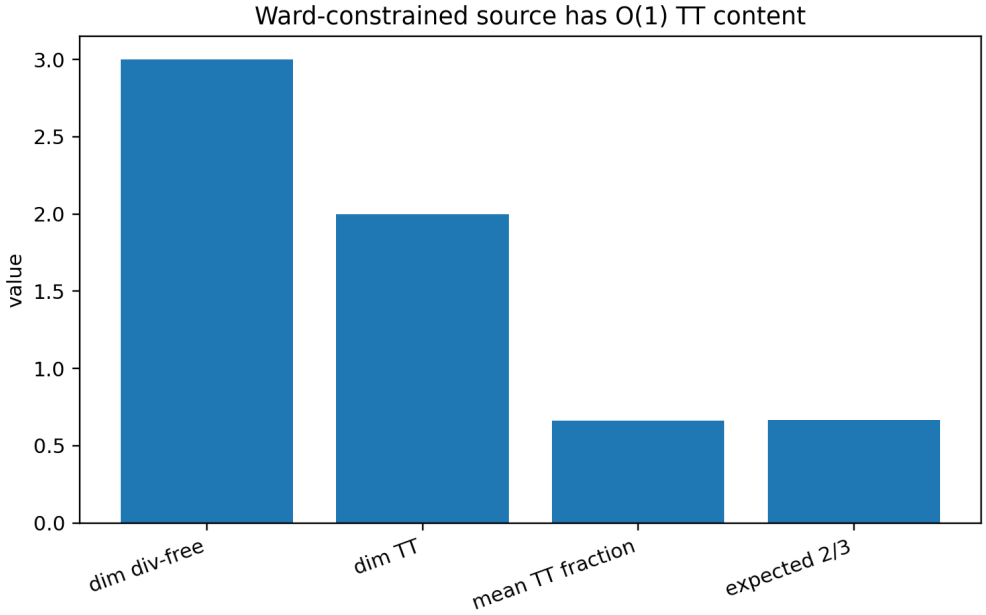

Figure 1: The divergence-free constrained stress subspace has dimension three and contains a two-dimensional TT subspace. Random constrained stresses have order-one TT content.

3 Response exponents

The audited low branch and source responses are:

Diagnostic	Exponent	Interpretation
Lowest carrier branch $\omega_0(k)$	0.999999942266	linear cone
Projected TT response	−2.000001654952	desired $1/k^2$ pole
Ward-constrained local stress response per TT	−2.000001654952	same pole, supplied by constraint
Beltrami local conserved-source proxy	1.999995874097	derivative suppressed
Cotton exact-local-TT proxy	3.999995441147	Cotton suppressed

The positive result is the equality of the projected and Ward-constrained response exponents. The local conservation constraint supplies the TT response with finite residue. The negative boundary is that a source operator whose conservation is achieved purely by local derivatives is necessarily soft in the infrared.

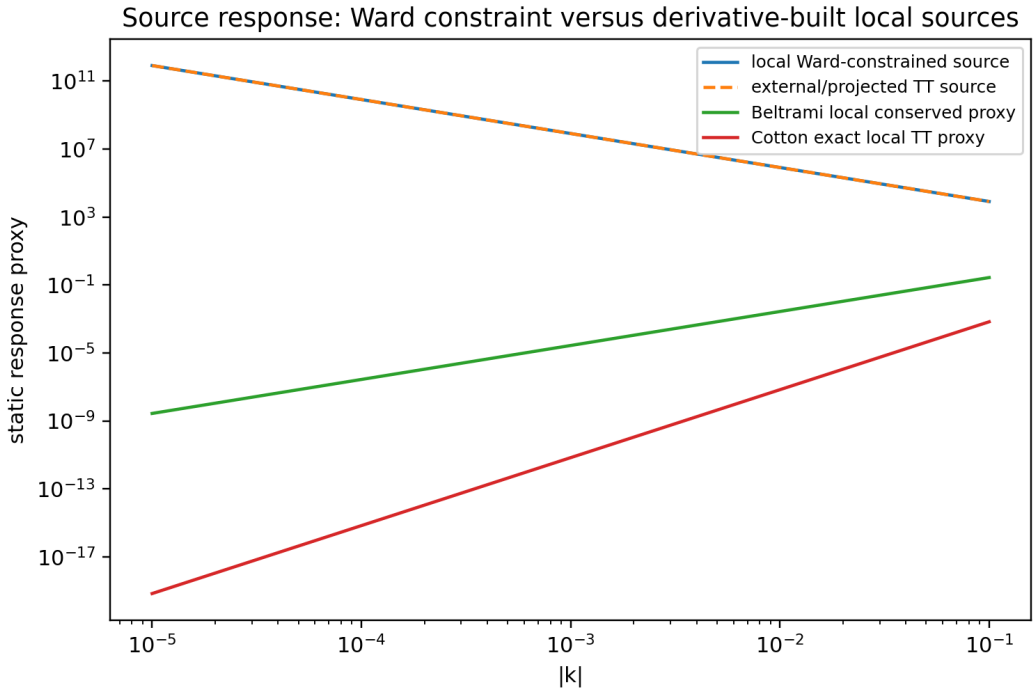

Figure 2: Projected/external TT sources and local Ward-constrained stress sources see the $1/k^2$ pole. Derivative-built local conserved/TT sources are suppressed.

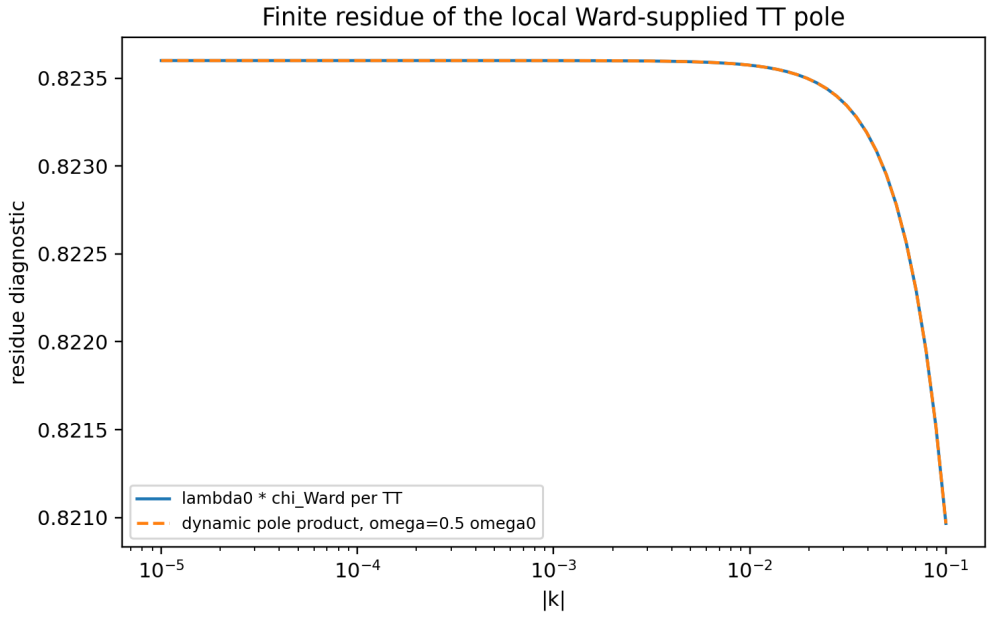

Figure 3: The local Ward-supplied pole has a finite residue diagnostic.

4 Why derivative-built local sources still fail

If one insists that the source operator itself be local and identically conserved as a polynomial tensor built only from local derivatives, then locality forces derivative factors. A Beltrami/double-curl construction is conserved, but starts at second derivative order; its response scales as k^2 after multiplication by the carrier pole. An exact local TT tensor starts at Cotton order, and its response scales as k^4.

This distinction matters. The local Ward-constrained stress sector says that the TT projection need not be inserted as a nonlocal source. But it does not yet provide a microscopic matter Hamiltonian whose stress tensor obeys the Ward identity dynamically. That is the next target.

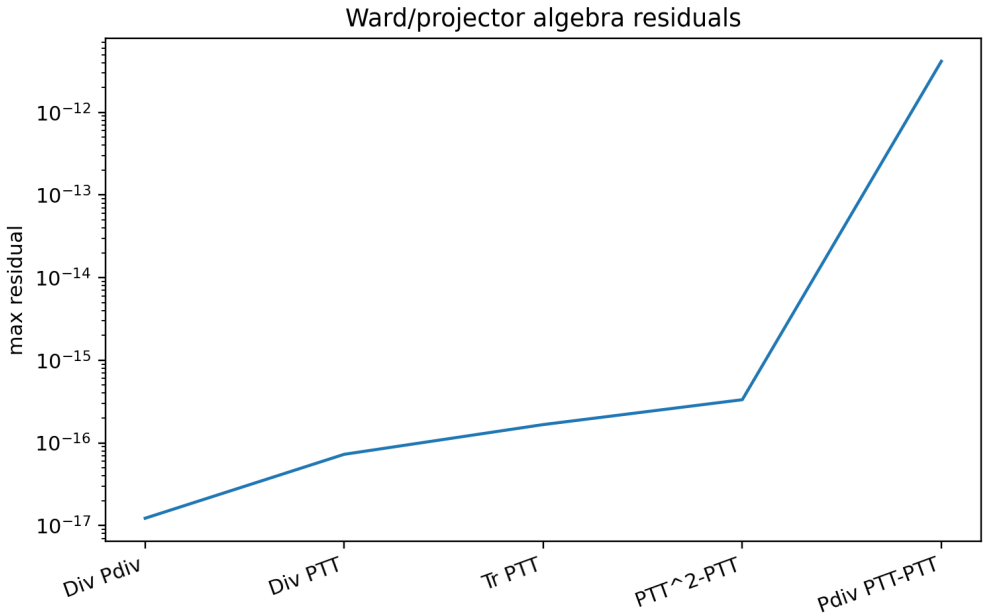

Figure 4: Ward/projector residuals are at numerical precision.

5 Verdict and next step

The carrier status improves modestly. The chiral bosonic spin-2 architecture can be coupled to a *local constrained stress sector* that supplies the same $1/k^2$ TT pole as an externally projected source. This is stronger than merely inserting P_{TT} by hand.

The result is still not a full matter-coupled graviton phase. The audit does not construct a microscopic local matter Hamiltonian, universal stress tensor, or nonlinear Ward identity. It identifies the next precise construction problem:

> Build local matter degrees of freedom whose Ward identity enforces $d_i T_{ij} = 0$ while retaining finite TT matrix elements.

If that succeeds, the carrier becomes a genuinely local matter-coupled linear spin-2 response model. If it fails, the obstruction can be stated as a local matter/Ward no-go rather than a carrier no-go.