CORRECTION AND SCOPE

Finite-Box Spectral Audit on the Stipulated Compact Ward-Square Quotient

Original title · Hard-Constraint Spectral Audit for the Compact Ward-Square Tensor Model

The spectra refer to the stipulated hard quotient. The harmonic target and finite-box spectral observations carry separate assumptions.

Manuscript · p. 1, spectral audit

Tower-Filtered ED Lanczos Scaling Audit for the Compact Ward-Square Spin-2 Box

Evidence · Reduced finite-spin tower scaling / Local hard-Gauss move obstruction

Hard-Constraint Spectral Audit for the Compact Ward-Square Tensor Model

QATNU numerical audit

July 3, 2026

Verdict

This audit replaces the soft-penalty compact screen by an exact hard-constraint spectral calculation. The local constraints

$$\nabla_i A^{(s)}_{ij} = 0, \quad \operatorname{tr} A^{(s)} = 0, \qquad \nabla_i (S_{ij} - A^{(1)}_{ij}) = 0, \quad \operatorname{tr}(S - A^{(1)}) = 0$$

are imposed by working directly in the local constraint null space. No TT projector is inserted as a microscopic interaction; the projection appears only after eliminating local Gauss/trace constraints.

The result is positive at the hard-constrained harmonic spectral-function level: the assembled tensor/matter Ward-square model has exactly two soft TT branches, $z \simeq 1$, a k^{-2} Ward-stress response, a finite next gap, and no extra non-origin zero in the Brillouin-zone scan. This is stronger than a soft-penalty screen, but it is still not a full nonperturbative finite-spin Hilbert-space proof.

Hard-constrained quotient

For each TT polarization the constraints can be parameterized by

$$R = S - A^{(1)}, \qquad A^{(1)}, \ldots, A^{(W)}.$$

Equivalently, $S = A^{(1)} + R$. The inherited kinetic metric is kept; frequencies are computed from the generalized problem $K_y v = \omega^2 G_y v$. The stiffness is

$$K(k) = L_{\text{path}} + \left(|n(k)|^2 + m_W(k)^2\right) \sum_{s=1}^{W} |s\rangle\langle s|,$$

with $W = 16$.

Infrared result

The deepest-infrared fit gives

$$z = 0.999493439, \qquad \chi_S(0,k) \sim k^{-1.998965499}.$$

The pole residue diagnostic is

$$\lambda_0(k)\chi_S(0,k) = 0.058823546,$$

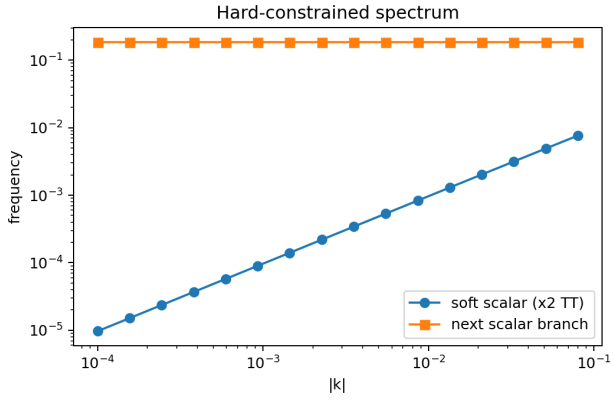
matching

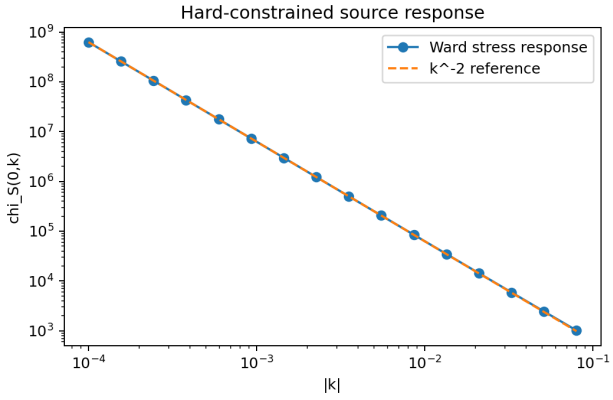
$$1/(W+1) = 0.058823529.$$

The next scalar branch gap at the smallest sampled momentum is

$$\Delta_2 = 0.184536719.$$

Tensoring with the two TT polarizations gives exactly two low branches.



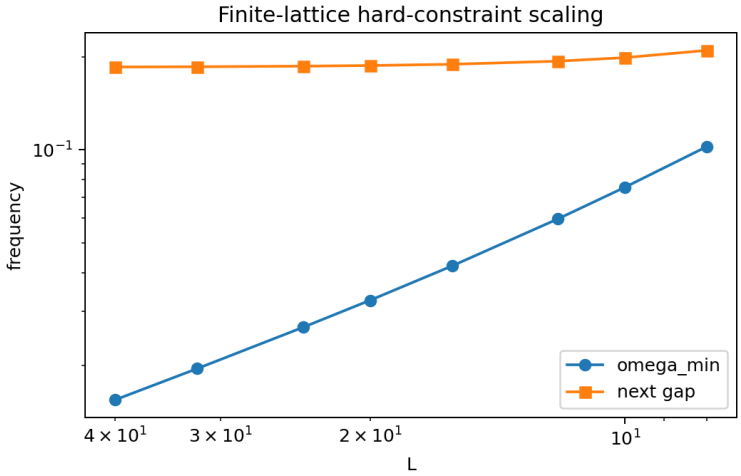


Finite-lattice scaling

L	$k_{\min}$	$\omega_{\min}$	next gap	$\chi_S(0,k)$	$\lambda\chi$
8	0.785398	1.024368e-01	0.209608	1.063067e+01	0.111551
10	0.628319	7.558846e-02	0.198559	1.563837e+01	0.089352
12	0.523599	5.969251e-02	0.193395	2.201756e+01	0.078453
16	0.392699	4.203932e-02	0.188980	3.894498e+01	0.068828
20	0.314159	3.253492e-02	0.187210	6.129570e+01	0.064883
24	0.261799	2.659468e-02	0.186327	8.892562e+01	0.062895
32	0.196350	1.954784e-02	0.185506	1.597278e+02	0.061035
40	0.157080	1.548741e-02	0.185146	2.510419e+02	0.060215

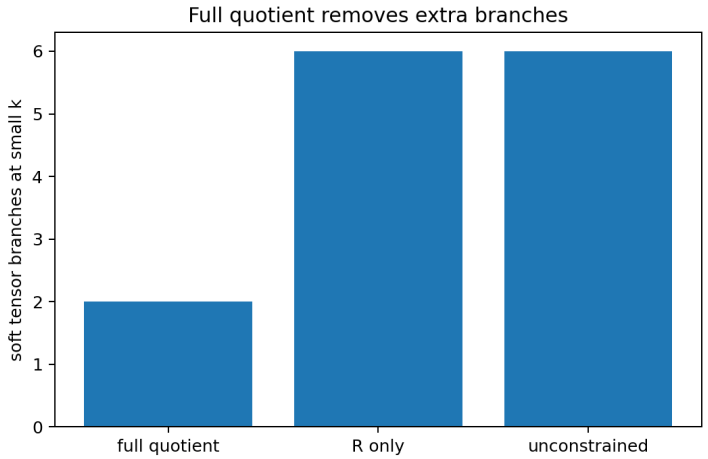
The finite-lattice fit gives

$$z = 1.116745246, \qquad \chi_S(0,k) \sim k^{-2.025248590}.$$



Full quotient check

At small nonzero momentum the full quotient has 2 tensor branches below 10^{-2}: the desired two TT branches. If only the covariant relative constraint is imposed, the expected low tensor count is 6; the full tensor quotient is therefore essential.



Brillouin-zone scan

A full scalar hard-constraint scan at $L = 13$ checked 2196 nonzero grid momenta. The minimum next scalar gap is

$$\min \Delta_2 = 0.191816426,$$

and the minimum low frequency outside $|k| > 0.75$ is

$$\min_{|k|>0.75} \omega_1 = 0.102709675.$$

The number of extra zero points below 10^{-8} was 0. At the known coined doubler,

$$\omega(k_D) = 0.639905158,$$

so the doubler is gapped in this hard-constrained Ward-square target.

Real-space rank law

The finite-lattice local constraints obey

$$\operatorname{rank} C = (W+1)(4V-3), \qquad \dim \ker C = (W+1)(2V+3).$$

L	W	rank	expected rank	null dim	expected null dim
2	1	58	58	38	38
2	2	87	87	57	57
2	3	116	116	76	76
3	1	210	210	114	114
3	2	315	315	171	171

Boundary

This hardens the evidence that the compact Ward-square tensor/matter construction has the correct constrained harmonic spectral function. It still does not prove the full finite-spin phase. The remaining obligations are nonperturbative: exact finite-spin gauge-generator implementation, bounded-flux saturation in the quantum Hilbert space, phase slips/defects in the fully constrained ensemble, and a direct many-body spectral function rather than a harmonic symbol.