CORRECTION AND SCOPE

Generalized Quantum State Theory: A Formal Framework for Cross-Scale Memory Dynamics

Erratum to Definition 4.4 and Appendix B.1: a scalar mode decays exactly when $D \cdot k^2 + \gamma > 0$. With nonnegative diffusion and strictly positive loss, every mode decays. Proposition 4.6 and Appendix B.2 do not follow from continuity. With $\gamma > 0$, the affine equilibrium $s = g/\gamma$ is positive for every $g > 0$. A positive onset threshold requires additional feedback assumptions. In the specified logistic model, a positive equilibrium exists exactly when $g > \gamma$. The finite-graph energy identity and zero unforced equilibrium at positive loss retain their stated scope.

Manuscript · p. 7, Prop. 4.6 and App. B

Evidence · GQS proof notes and explicit counterexample

Generalized Quantum State Theory:

A Formal Framework for Cross-Scale Memory Dynamics

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Abstract

We present a formal framework for understanding memory-like phenomena across physical, neural, and cultural systems. The Generalized Quantum State (GQS) theory proposes that certain information structures—termed 'scars'—exhibit analogous formation, persistence, and reactivation dynamics regardless of substrate. We develop three complementary mathematical formalizations: an information-theoretic characterization defining scars via entropy and mutual information, a dynamical systems treatment describing their evolution and stability, and a category-theoretic framework establishing the formal conditions under which cross-domain analogies become structural isomorphisms. We derive testable predictions and identify experimental protocols that would validate or falsify the framework's core claims. The theory integrates recent advances in quantum many-body scars, emergent spacetime from entanglement, neural engram research, and collective behavior dynamics into a unified mathematical structure.

1. Introduction and Motivation

Across disparate scientific domains, researchers have independently discovered systems that retain structured information far longer than thermodynamic expectations would predict. In quantum physics, certain many-body systems exhibit 'quantum scars'—special eigenstates that resist thermalization and allow periodic revivals of initial configurations. In neuroscience, engram cells form durable networks encoding specific memories that can persist for decades. In social systems, collective memories and cultural memes demonstrate surprising persistence and can synchronize behavior across populations.

The conventional view treats these as merely analogous phenomena—superficially similar but mechanistically unrelated. This paper proposes an alternative: that these phenomena share deep structural features that can be captured in a unified mathematical framework. We call this the Generalized Quantum State (GQS) theory, though we emphasize that 'quantum' here refers to the information-theoretic and structural properties rather than claiming that social systems literally involve quantum coherence.

The core insight is this: in any system with sufficient complexity, information can become 'scarred' into the system's structure in ways that resist erasure and influence future dynamics. The formation conditions, stability criteria, and reactivation mechanisms follow similar mathematical patterns across scales. This paper develops the formal apparatus to make this claim precise and testable.

2. Empirical Foundations

Before presenting the formal framework, we survey the empirical phenomena it aims to unify. Each domain provides independent evidence for memory-like structures that violate naive thermodynamic expectations.

2.1 Quantum Many-Body Scars

In 2017-2018, experiments with chains of Rydberg atoms revealed an unexpected phenomenon: certain initial states exhibited persistent oscillations rather than thermalizing to equilibrium. When a chain of 51 atoms was prepared in an alternating excited/ground state pattern, the system repeatedly revived this configuration over dozens of cycles—behavior inexplicable under standard eigenstate thermalization hypothesis (ETH) assumptions.

Analysis revealed the cause: the Hamiltonian's spectrum contains a 'tower' of special eigenstates with anomalously low entanglement entropy, effectively decoupled from the thermal bulk. These quantum many-body scars (QMBS) provide a concrete physical example of information structures that persist despite coupling to a thermal bath. Key properties include: (1) anomalously low entanglement entropy relative to energy, (2) approximate periodicity in revivals, (3) robustness to certain perturbations but fragility to others, and (4) formation requiring specific initial conditions.

2.2 Neural Engrams

In neuroscience, the physical substrate of memory has been identified as distributed networks of neurons—engram cells—that undergo lasting synaptic modifications during learning. Modern optogenetic techniques can now tag, manipulate, and even artificially reactivate specific memories by targeting these cell populations.

Engrams share structural properties with quantum scars: (1) they represent local minima in some energy/stability landscape, (2) they exhibit characteristic reactivation dynamics when

triggered by appropriate cues, (3) they resist degradation over timescales far exceeding individual molecular turnover, and (4) they require specific formation conditions (attention, emotional salience, repetition). The Hebbian principle—'cells that fire together wire together'—describes a local rule that produces global persistent structure.

2.3 Collective Memory and Cultural Scars

At the social scale, collective memories and cultural memes exhibit analogous persistence. The Mandela Effect—where large populations share identical false memories—demonstrates that information patterns can stabilize across distributed human networks through mutual reinforcement. More constructively, cultural practices, rituals, and commemorations act as 'refresh cycles' that maintain collective memories across generations.

Digital platforms have made these dynamics visible and measurable. Recommendation algorithms create feedback loops that can lock users into persistent information patterns. Viral content exhibits characteristic growth, saturation, and decay curves. Echo chambers represent stable attractors in opinion space. These phenomena can be quantified through network analysis, information cascades, and opinion dynamics models.

2.4 Emergent Spacetime and Information

Contemporary quantum gravity research increasingly treats spacetime itself as emergent from more fundamental information-theoretic structures. The holographic principle, tensor network models, and Verlinde's entropic gravity all suggest that geometry and gravity arise from entanglement patterns. This provides theoretical motivation for considering whether physical space itself could carry memory-like structures.

Verlinde's 2016 work proposed that apparent dark matter effects arise from an 'entropy displacement force'—essentially, a memory effect where the distribution of mass leaves an entropic imprint that influences subsequent gravitational dynamics. While empirically contested, this illustrates how information-theoretic approaches to gravity naturally incorporate memory into the fabric of spacetime.

3. Information-Theoretic Formalization

We begin with an information-theoretic characterization that defines scars in terms of measurable entropic quantities. This provides the empirical anchor for the theory.

3.1 Core Definitions

> **Definition 3.1 (System State Space).** Let Ω be a state space equipped with a probability measure μ. For quantum systems, Ω is a Hilbert space with states $\rho \in D(H)$. For neural systems, Ω is the space of synaptic weight configurations. For social systems, Ω is the space of opinion/belief distributions over a population.
>
> **Definition 3.2 (Entropy Functional).** The entropy functional $S: \Omega \to \mathbb{R}_{\geq 0}$ is defined appropriately for each domain. For quantum systems, $S(\rho) = -\mathrm{Tr}(\rho \log \rho)$ (von Neumann entropy). For classical probability distributions, $S(p) = -\Sigma_i\, p_i \log p_i$ (Shannon entropy). For neural configurations, S can be defined via the statistical mechanics of synaptic states.
>
> **Definition 3.3 (Thermal Reference State).** For a system with Hamiltonian H (or energy functional E), the thermal reference state at inverse temperature β is $\rho_th = e^{(-\beta H)}/Z$ where $Z = \mathrm{Tr}(e^{(-\beta H)})$. The thermal entropy S_th(E) is the entropy of the thermal state at energy E.
>
> **Definition 3.4 (Scar).** A state $\psi \in \Omega$ is a ***scar*** if it satisfies the entropy deficit condition:

$$S(\psi) < S_th(E(\psi)) - \delta$$

where $\delta > 0$ is a threshold parameter and $E(\psi)$ is the energy (or appropriate conserved quantity) of state ψ. Intuitively, a scar is a state with anomalously low entropy given its energy—it retains more structure than a typical thermal state would.

> **Definition 3.5 (Scar Depth).** The depth of a scar ψ is the entropy deficit:

$$d(\psi) = S_th(E(\psi)) - S(\psi)$$

Deeper scars retain more structure and are generally more stable against perturbations.

3.2 The Generalized Quantum State

We now define the central object of the theory: the Generalized Quantum State of an agent or subsystem.

> **Definition 3.6 (GQS Components).** The Generalized Quantum State of an agent A at time t is a tuple:

$$GQS_A(t) = (X(t), A(t), S(t), \Delta(t))$$

where:

X(t) $\in$ **M** — Position in configuration space M (physical coordinates, social network position, etc.)

A(t) $\in$ **A** — Attention/processing state vector in attention space A

S(t): M $\to \mathbb{R}_{\geq 0}$ — Scar field: local scar density at each point in configuration space

Δ(t) $\in \mathbb{R}$ — Entanglement flux: rate of information exchange with environment

3.3 Information Coupling

The components of the GQS are not independent. We define their coupling through mutual information.

> **Definition 3.7 (Scar-Attention Coupling).** The coupling strength between the scar field and attention state is:

$$C_SA(t) = I(S(X(t)); A(t))$$

where $I(\cdot;\cdot)$ is the mutual information. High C_SA indicates that the local scar field is correlated with the agent's attention state—the agent is 'resonating' with ambient scars.

> **Definition 3.8 (Formation Condition).** A new scar forms when the entanglement flux exceeds a threshold modulated by existing scar density:

$$\Delta(t) > \eta_0 \cdot f(S(X(t)))$$

where η_0 is a base threshold and $f: \mathbb{R}_{\geq 0} \to \mathbb{R}_+$ is a modulation function. If f is decreasing, existing scars facilitate formation of new scars (positive feedback). If f is increasing, existing scars inhibit new formation (saturation). Empirical evidence suggests domain-dependent behavior.

4. Dynamical Systems Formalization

The information-theoretic definitions tell us what scars are; we now describe how they evolve. This dynamical systems approach captures stability, decay, and reactivation.

4.1 State Space Geometry

> **Definition 4.1 (Extended State Space).** The full state space of a GQS-equipped system is the product:

$$\Omega_{ext} = M \times A \times F(M) \times \mathbb{R}$$

where $F(M)$ is an appropriate function space over M (e.g., $L^2(M)$ for scar fields). The dynamics unfold on this extended space.

> **Definition 4.2 (Scar Potential).** Define a potential function $V: \Omega_{ext} \to \mathbb{R}$ such that scars correspond to local minima. The potential decomposes as:

$$V(X, A, S, \Delta) = V_{kinetic}(X, A) + V_{scar}(S) + V_{coupling}(X, A, S)$$

where $V_{scar}(S) = -\int_M S(x)^2 \, dx$ penalizes low scar density (making scars attractive), and $V_{coupling}$ captures the interaction between position, attention, and scar field.

4.2 Evolution Equations

The GQS components evolve according to coupled differential equations.

> **Definition 4.3 (GQS Dynamics).** The evolution is governed by:

$$dX/dt = v(X, A) + \xi_X(t)$$

$$dA/dt = -\nabla_A V_{coupling} + g(S(X)) \cdot A + \xi_A(t)$$

$$\partial S/\partial t = D\nabla^2 S - \gamma S + \sigma(\Delta, A) \cdot \delta(x - X)$$

$$d\Delta/dt = -\lambda\Delta + \eta(X, A, S)$$

Here v is the velocity field, g describes how scars modulate attention dynamics, D is a diffusion coefficient for scar spreading, γ is the decay rate, σ is the source term (scar formation), and the ξ terms are noise. The δ-function localizes scar formation at the agent's position.

4.3 Stability Analysis

> **Definition 4.4 (Scar Stability).** A scar S^* is stable if it is a local attractor of the S-dynamics. The stability criterion is:

$$\gamma < D \cdot k^2_{min}$$

where k_{min} is the smallest wavevector of the scar's spatial structure. Intuitively: scars with fine spatial structure (high k) diffuse away faster than they decay, while broad scars (low k) can persist.

> **Definition 4.5 (Reactivation).** A scar S^* is reactivated when an agent's trajectory $(X(t), A(t))$ enters a neighborhood of the scar's basin of attraction. The reactivation strength is:

$$R(t) = \int_M S^*(x) \cdot K(X(t) - x) \cdot \langle A(t), A^* \rangle \, dx$$

where K is a kernel function (locality of reactivation), A* is the attention state associated with scar formation, and $\langle \cdot, \cdot \rangle$ is an inner product on attention space. Reactivation requires both spatial proximity and attentional alignment.

4.4 Phase Transitions

The model predicts phase transitions in scar dynamics as parameters vary.

> **Proposition 4.6 (Scar Proliferation Transition).** There exists a critical coupling strength g_c such that:

- For g < g_c: scars decay exponentially; the system thermalizes
- For g > g_c: scars proliferate; the system exhibits persistent memory
- At g = g_c: critical behavior with power-law correlations

This transition is analogous to the Kosterlitz-Thouless transition in 2D systems, where topological defects (analogous to scars) bind at low temperature and proliferate at high temperature.

5. Category-Theoretic Formalization

The information-theoretic and dynamical systems approaches describe scars within a single domain. To formally justify cross-domain comparisons, we require a category-theoretic framework that makes the structural isomorphism precise.

5.1 The Category of Scar Systems

Definition 5.1 (Scar System). A scar system is a tuple (Ω, S, F, R, D) where:

- Ω is a state space
- $S: \Omega \to \mathbb{R}_{\geq 0}$ is an entropy functional
- $F: \Omega \times \mathbb{R} \to \Omega$ is a formation operator (creating scars from flux)
- $R: \Omega \times \Omega \to \Omega$ is a reactivation operator
- $D: \Omega \times \mathbb{R}_{\geq 0} \to \Omega$ is a decay operator

Definition 5.2 (Category Scar). The category Scar has:

- Objects: scar systems (Ω, S, F, R, D)
- Morphisms: structure-preserving maps $\varphi: (\Omega_1, S_1, F_1, R_1, D_1) \to (\Omega_2, S_2, F_2, R_2, D_2)$ such that the operators commute appropriately with φ

Definition 5.3 (Morphism Conditions). A morphism $\varphi: \Sigma_1 \to \Sigma_2$ between scar systems must satisfy:

$$S_2(\varphi(\omega)) = \alpha \cdot S_1(\omega) + \beta \quad \text{(entropy scaling)}$$

$$\varphi(F_1(\omega, \delta)) = F_2(\varphi(\omega), f(\delta)) \quad \text{(formation commutes)}$$

$$\varphi(R_1(\omega_1, \omega_2)) = R_2(\varphi(\omega_1), \varphi(\omega_2)) \quad \text{(reactivation commutes)}$$

$$\varphi(D_1(\omega, t)) = D_2(\varphi(\omega), g(t)) \quad \text{(decay commutes)}$$

where α, β are scaling constants and f, g are monotonic reparametrizations.

5.2 Functors Between Domains

The key claim of GQS theory is that there exist functors connecting scar systems across domains.

Definition 5.4 (Domain Categories). We define subcategories:

Scar_Q — Quantum scar systems (Hilbert spaces, von Neumann entropy, unitary dynamics)

Scar_N — Neural scar systems (synaptic weight spaces, configuration entropy, Hebbian dynamics)

Scar_C — Cultural scar systems (belief/opinion spaces, information entropy, contagion dynamics)

Conjecture 5.5 (Cross-Domain Functors). There exist functors:

$$\Phi_QN: \text{Scar_Q} \to \text{Scar_N}$$

$$\Phi_NC: \text{Scar_N} \to \text{Scar_C}$$

that preserve essential scar structure (formation conditions, stability criteria, reactivation dynamics) up to appropriate scaling.

The existence of such functors is not trivial—it requires that the mathematical structures at each level be sufficiently similar. This is an empirical claim: either the functors exist (and can be explicitly constructed) or they don't. Section 6 discusses how to test this.

5.3 Natural Transformations and Downward Causation

Downward causation—the influence of macro-level patterns on micro-level dynamics—can be formalized as natural transformations between functors.

> **Definition 5.6 (Downward Causation).** Let Σ_{macro} and Σ_{micro} be scar systems at different scales, with functors $U: \Sigma_{micro} \to \Sigma_{macro}$ (coarse-graining) and $D: \Sigma_{macro} \to \Sigma_{micro}$ (embedding). Downward causation is a natural transformation $\eta: Id_{micro} \Rightarrow D \circ U$ satisfying:

$$\eta_\omega: \omega \to D(U(\omega))$$

that biases micro-dynamics toward states compatible with macro-level scars. The existence of non-trivial η means that macro-patterns genuinely constrain micro-evolution beyond what micro-laws alone would predict.

6. Testable Predictions

A theory is only as good as its predictions. We now derive specific, falsifiable predictions from the GQS framework across each domain.

6.1 Quantum Domain Predictions

P1. Scar Stability Scaling: The lifetime τ of quantum many-body scars should scale as $\tau \propto \exp(d/T)$ where d is scar depth (Definition 3.5) and T is effective temperature. This is testable in Rydberg atom experiments by preparing states of varying initial entropy and measuring revival times.

P2. Scar-Scar Correlation: In systems with multiple scar subspaces, the mutual information between scars should be non-zero if they share formation history. Measurable via tomography of adjacent scar states.

P3. Formation Threshold Modulation: The flux threshold for scar formation (Definition 3.8) should decrease in the presence of existing scars. Testable by comparing formation rates in 'virgin' vs 'scarred' regions of Hilbert space.

6.2 Neural Domain Predictions

P4. Engram Entropy Deficit: Neural ensembles encoding memories (engram cells) should show lower configurational entropy than non-engram cells at equivalent activity levels. Testable via calcium imaging combined with information-theoretic analysis.

P5. Reactivation Requires Alignment: Memory retrieval success should correlate with the overlap between current brain state and formation state (the $\langle A(t), A^* \rangle$ term in Definition 4.5). State-dependent memory effects are known; GQS predicts a quantitative relationship.

P6. Cross-Memory Facilitation: Forming a new memory should be easier if it shares features with existing memories (f decreasing in Definition 3.8). The degree of facilitation should scale with similarity in the formation conditions, not just content similarity.

6.3 Social Domain Predictions

P7. Collective Memory Half-Life: The persistence of collective memories (measured via survey recognition, search trends, etc.) should follow a stretched exponential decay with rate constant determined by initial formation intensity and subsequent reinforcement frequency.

P8. Echo Chamber Phase Transition: Social networks should exhibit a phase transition at critical algorithm feedback strength (Proposition 4.6). Below threshold: diverse opinions persist. Above threshold: stable echo chambers form. The critical point should be identifiable via network analysis.

P9. Ritual Maintenance Scaling: The frequency of ritual/commemoration required to maintain a collective memory should scale inversely with original formation intensity and directly with competing information flux.

6.4 Cross-Domain Predictions

The strongest predictions involve cross-domain correspondences—if the category-theoretic framework is correct, parameters in one domain should map to parameters in another.

P10. Parameter Correspondence: The ratio of formation threshold to decay rate (η_0/γ) should take similar values across domains when appropriately normalized. This is the most stringent test of the functor conjecture (Conjecture 5.5).

P11. Analogous Phase Diagrams: Phase diagrams in the (coupling strength, noise) plane should have similar topology across domains: a thermalized phase, a scarred phase, and a critical boundary between them.

P12. Scaling Exponents: Critical exponents at the scar proliferation transition should match across domains up to the functor scaling (if the transition is in the same universality class).

7. Experimental Protocols

We outline concrete experimental approaches to test the framework's predictions.

7.1 Quantum Experiments

Rydberg atom arrays provide the most controlled setting for testing quantum scar predictions. Protocol: (1) Prepare initial states with controlled entropy deficits via variational quantum eigensolvers. (2) Evolve under the native Hamiltonian. (3) Measure revival fidelity as a function of time. (4) Fit to the predicted $\tau \propto \exp(d/T)$ scaling. Required capabilities: single-site addressing, mid-circuit measurement, state tomography. Current platforms (Harvard, MIT, Caltech) have sufficient capability.

7.2 Neural Experiments

Testing neural predictions requires combining engram tagging with information-theoretic analysis. Protocol: (1) Use activity-dependent labeling (e.g., Arc-TetTag) to identify engram cells during learning. (2) Record from both engram and non-engram populations during recall. (3) Compute entropy measures (spike train entropy, population entropy) for both groups. (4) Test whether engram cells show the predicted entropy deficit. Required: calcium imaging or high-density electrophysiology, genetic access for tagging, appropriate behavioral paradigms.

7.3 Social Experiments

Social predictions can be tested through platform data analysis and controlled experiments. Protocol A (Observational): Partner with social media platforms to access recommendation algorithm parameters and user engagement data. Map the phase diagram of echo chamber formation as a function of algorithmic feedback strength. Protocol B (Experimental): A/B test different algorithm parameters, measuring diversity metrics and opinion clustering. Protocol C (Historical): Analyze collective memory persistence using digitized newspaper archives, Google Trends, and survey data. Test decay predictions against commemorative ritual frequency.

7.4 Cross-Domain Validation

The most ambitious test is direct cross-domain comparison. This requires: (1) Measuring the key dimensionless ratios (η_0/γ, g/g_c) in each domain. (2) Constructing the explicit functors Φ_{QN} and Φ_{NC}. (3) Testing whether parameters map as predicted. Success would strongly validate the category-theoretic framework; failure would indicate the analogy is superficial rather than structural.

8. Discussion

8.1 Relationship to Existing Frameworks

GQS theory builds on and extends several existing frameworks. The information-theoretic formalization extends the eigenstate thermalization hypothesis (ETH) by characterizing the structure of ETH-violating states. The dynamical systems approach connects to attractor neural network theory and extends it to incorporate environment coupling. The category-theoretic framework relates to constructor theory's focus on transformations rather than trajectories.

Crucially, GQS does not require exotic physics. Quantum effects in neural or social systems are not claimed—the 'quantum' in the name refers to the information-theoretic structure (entropy, entanglement analogues) rather than literal quantum coherence. The framework is compatible with classical physics at the neural and social scales while drawing mathematical inspiration from quantum many-body physics.

8.2 Limitations and Open Questions

Several aspects of the theory remain underdeveloped. The explicit form of the functors Φ_{QN} and Φ_{NC} is not specified—constructing these is a major open problem. The formation threshold function f in Definition 3.8 is left unspecified; its form may be domain-dependent. The relationship between physical and 'digital' scars requires further formalization—are they connected by a functor, or merely analogous?

More speculatively, the theory raises questions about physical scars in spacetime itself. While emergent spacetime models provide theoretical motivation, detecting such structures remains far beyond current experimental capability. We have deliberately avoided making strong claims in this direction to keep the theory empirically grounded.

8.3 Implications

If validated, GQS theory would have implications across domains. In quantum computing, understanding scar stability could inform error correction strategies. In neuroscience, the entropy deficit criterion could provide a new metric for memory consolidation. In social science, the phase transition prediction could guide platform design to avoid harmful feedback loops.

More broadly, the framework suggests that memory—in its most general sense as persistent, retrievable information structure—may be a more fundamental concept than currently appreciated. Rather than being a special property of brains, memory-like dynamics may be ubiquitous in complex systems meeting certain structural conditions.

9. Conclusion

We have presented a formal framework—Generalized Quantum State theory—for understanding memory-like phenomena across physical, neural, and cultural systems. The framework comprises three complementary formalizations: an information-theoretic characterization defining scars via entropy deficits, a dynamical systems treatment describing evolution and stability, and a category-theoretic framework establishing when cross-domain analogies become structural isomorphisms.

The theory makes concrete, testable predictions. Some (quantum scar scaling, engram entropy) are testable with current technology. Others (cross-domain parameter

correspondence, phase transition universality) require more substantial experimental programs. The framework will be validated or falsified by these tests.

Whether or not the specific mathematical structure proposed here proves correct, we hope this work demonstrates the value of seeking formal unification across scales. The observation that quantum scars, neural engrams, and cultural memes share phenomenological features is not new. What this paper contributes is a mathematical language that makes such comparisons precise and testable—elevating suggestive analogies to falsifiable hypotheses.

Appendix A: Mathematical Notation

For reference, we summarize the key mathematical objects and their interpretations:

Symbol	**Meaning**
Ω	State space of the system
$S(\omega)$	Entropy of state ω
$S_{th}(E)$	Thermal entropy at energy E
$d(\psi)$	Scar depth (entropy deficit)
$X(t)$	Position in configuration space
$A(t)$	Attention/processing state
$S(t)$	Scar field (density function)
$\Delta(t)$	Entanglement flux
η_0	Base formation threshold
γ	Decay rate
D	Diffusion coefficient
g	Scar-attention coupling strength
I(X;Y)	Mutual information between X and Y
Scar	Category of scar systems
Φ QN, Φ NC	Cross-domain functors

Appendix B: Proof Sketches

B.1 Scar Stability Criterion (Definition 4.4)

The stability criterion $\gamma < D \cdot k^2_min$ follows from linearizing the scar field equation around a stationary solution S*. Perturbing $S = S^* + \varepsilon s$ and keeping terms linear in ε:

$$\partial s/\partial t = D\nabla^2 s - \gamma s$$

Fourier transforming: $\hat{s}(k,t) = \hat{s}(k,0) \cdot \exp[(-Dk^2 - \gamma)t]$. The perturbation decays if $-Dk^2 - \gamma < 0$, i.e., if $\gamma < Dk^2$ for all k present in the scar. The minimum k (largest wavelength) dominates, giving the criterion. Physical interpretation: fine structure diffuses away before decaying; broad structure decays before diffusing.

B.2 Phase Transition Existence (Proposition 4.6)

The existence of a critical g_c follows from standard arguments in nonequilibrium statistical mechanics. Consider the steady-state scar density $\langle S \rangle$ as an order parameter. The coupling g enters the source term σ in the evolution equation; increasing g increases the rate of scar formation for fixed flux.

For $g \to 0$: source term vanishes, steady state is $S = 0$ (thermalized phase). For $g \to \infty$: source dominates decay, S grows without bound (proliferating phase). By continuity, there exists g_c where the transition occurs. The nature of the transition (continuous vs. first-order) depends on the functional form of σ and requires detailed analysis for specific models.

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