Gauged/Ward-Completed Dynamic Matter Audit for the Chiral Spin-2 Carrier

QATNU numerical audit

Abstract

We test whether the local Ward stress source from the previous audit can be made dynamical without generating the relevant mass self-energy. A naive non-derivative coupling hS requires a tuned counterterm. Here the coupling is completed as a local positive covariant square,

$$H_m = \frac{\Omega^2}{2}\|S - \alpha h_{\rm wall}\|^2,$$

where S is a local divergence-free stress variable and $h_{\rm wall}$ is the lower-wall component of the chiral spin-2 carrier. The square fixes the seagull term and is protected at $k = 0$ by a diagonal shift symmetry. The resulting effective CV/Gaussian model has two linear TT branches, finite stress-source residue, and static response $\chi_S(0, k) \sim k^{-2}$. Generic unprotected first-class-local wall terms still gap the pole, so the result is a positive Ward-completed effective matter sector, not yet a microscopic finite-dimensional matter phase.

1 Construction

The chiral scalar carrier is

$$K_{\rm ch}(k) = D(k)^T D(k) + |n(k)|^2 I_W,$$

with the same local coined connection vector $n(k)$ and synthetic chiral factor used in the bosonic lift audit. At the origin D_0 has a normalized lower-wall null vector

$$v_0 \propto (1, r_0, r_0^2, \ldots, r_0^{W-1}).$$

The two spin-2 polarizations are represented by two copies of this scalar carrier.

The matter sector is a local symmetric stress variable constrained by

$$d_i S_{ij} = 0.$$

In the divergence-free Ward frame it has two TT components and one transverse scalar component. The dynamical coupling tested here is

$$H_m = \frac{1}{2}\Pi_S^2 + \frac{1}{2}(S - Cq_0)^T M (S - Cq_0),$$

where q_0 is the lower-wall coordinate of the two TT carrier copies, $Cq_0 = (\alpha q_{0,1}, \alpha q_{0,2}, 0)$, and $M > 0$ is local. The base run uses $M = \Omega^2 I$.

At $k = 0$ the coupled stiffness is invariant under

$$q_a \mapsto q_a + \epsilon v_0, \qquad S_a \mapsto S_a + \alpha v_0(0)\epsilon,$$

for each TT polarization $a = 1, 2$. This diagonal shift fixes the seagull term. Expanding the square gives the non-derivative source coupling and its Ward-required counterterm at the same time.

2 Main numerical checks

quantity	value
lowest branch exponent z	0.999606677871
second branch exponent z	0.999541049078
TT stress response exponent	-1.999574909721
random Ward stress response exponent	-1.999575423025
low positive TT branch count at $\lvert k\rvert = 10^{-3}$	2
first gapped origin eigenvalue	0.338173473027

Table 1: The Ward-completed dynamical stress sector keeps exactly two low TT branches and gives a $1/k^2$ stress-source response.

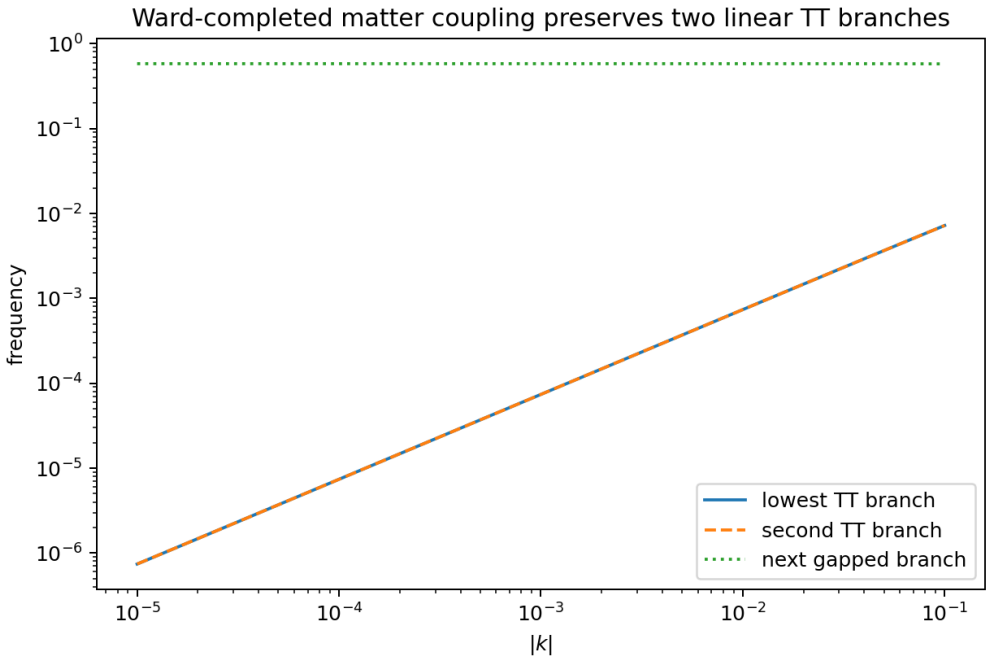

Figure 1: Two TT branches remain linear while the next branch is gapped.

3 Ward geometry and finite residue

The Ward frame residuals at $|k| = 10^{-3}$ are

$$\|DT_{TT}\|_\infty = 2.800e-17, \qquad \|\mathrm{tr}\, T_{TT}\|_\infty = 5.551e-17.$$

A random divergence-free stress has mean TT fraction

$$0.666936,$$

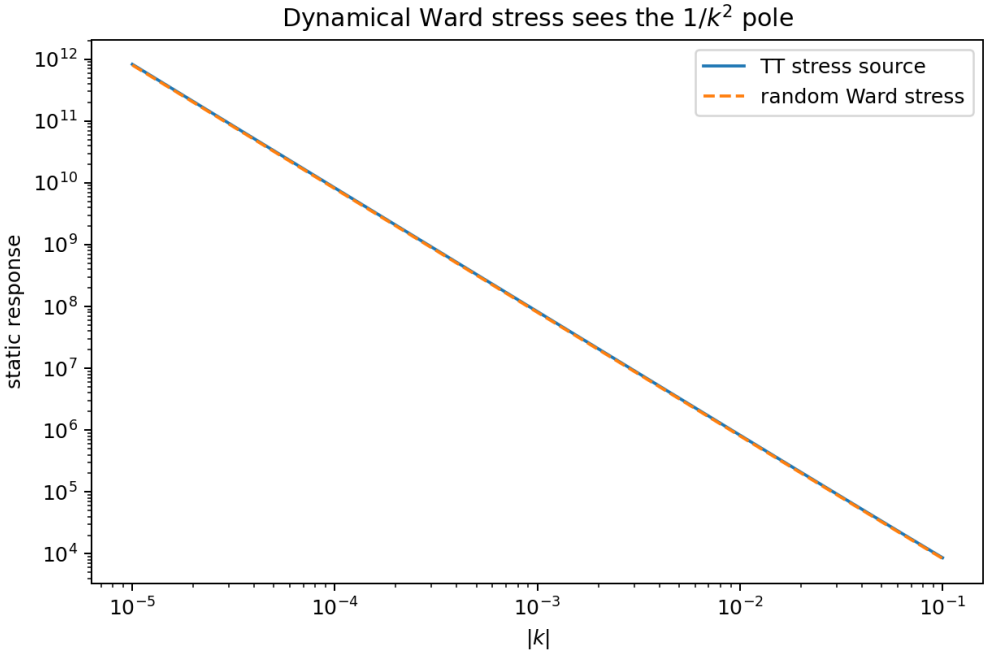

Figure 2: A source coupled to the dynamical Ward stress variables sees the static $1/k^2$ pole.

consistent with the expected 2/3 dimension ratio. Thus the source residue is not Cotton suppressed: it is an order-one projection of a local Ward-constrained stress variable.

The origin null-vector checks give

$$\|D_0 v_0\| = 1.564e-17, \qquad \|K_0 z_a\| = (1.122e-16, 1.122e-16),$$

with $v_0(0)^2 = 0.823600000000$.

4 Seagull and protection tests

The previous dynamical matter obstruction was caused by a non-derivative coupling without a symmetry-fixed seagull. Detuning the seagull here reproduces that failure. Overtuning produces a mass gap with

$$\Delta(\epsilon) \sim \epsilon^{0.493769},$$

while undertuning is unstable. The unique Ward-completed coefficient is the one preserving the diagonal shift.

Random local positive covariant-square perturbations that preserve the diagonal shift passed 64/64 stability tests, with

$$z \in [0.999732, 0.999867], \qquad \alpha_\chi \in [-1.999557, -1.999535].$$

By contrast, generic local positive wall terms that do not preserve the shift gap the pole, with gap range

$$[1.773e-02, 2.767e-01].$$

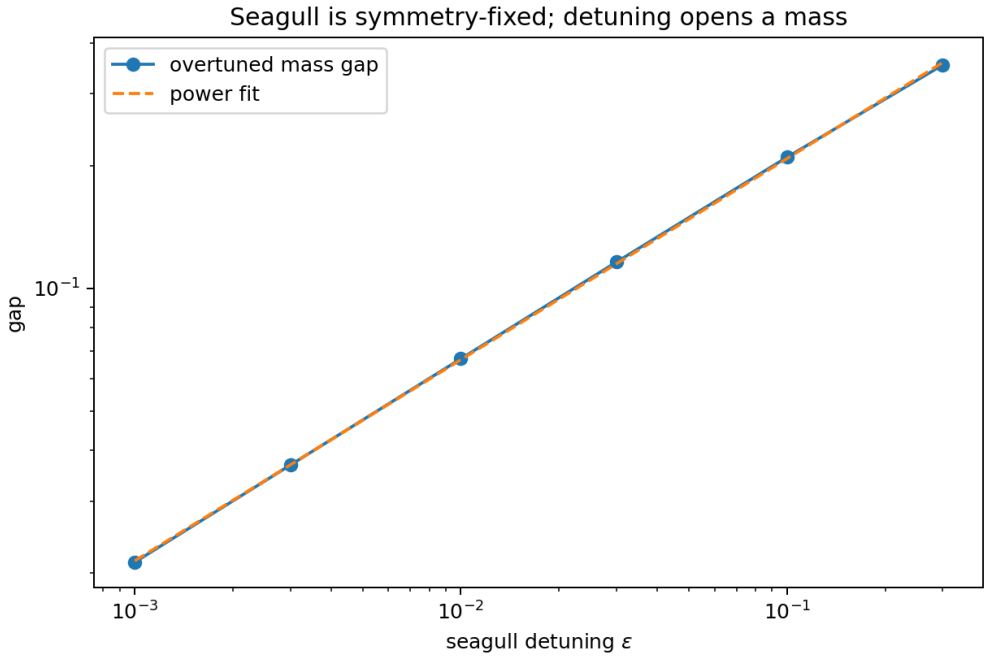

Figure 3: Detuning the Ward seagull opens the expected square-root mass gap.

5 Interpretation

The audit is positive for the immediate effective construction: the source coupling, mass counterterm, and finite residue are unified by a local Ward-completed square. The mechanism is the analogue of a diamagnetic/seagull Ward completion: the same local invariant that gives the non-derivative stress coupling also fixes the h^2 term that cancels the static self-energy.

The boundary remains important. This is not a full microscopic matter model and not an exact fixed-local-dimension phase. It is a local continuous-variable Gaussian/Ward sector showing the required symmetry structure. The next finite-dimensional route should realize this diagonal-shift/covariant-square mechanism with finite-spin, rotor, or topological-order variables rather than by an onsite cutoff guard.

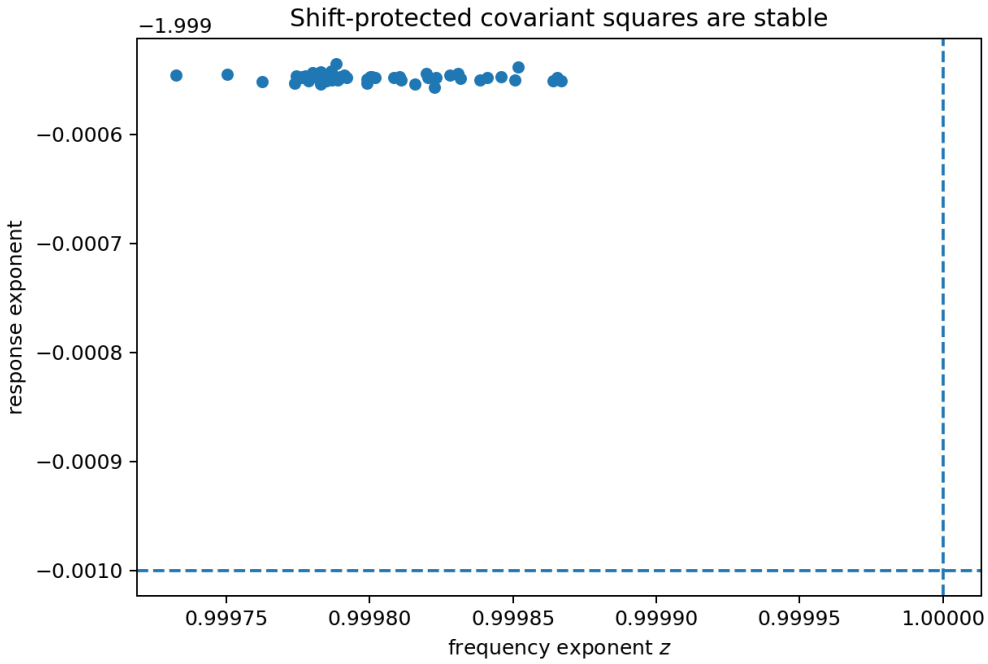

Figure 4: Shift-preserving covariant squares retain $z \simeq 1$ and $\chi \sim k^{-2}$.