CORRECTION AND SCOPE

Finite-Spin Ward-Square Implementation and Calibration

Original title · Finite-Spin Rotor Ward-Square Implementation Audit

Implementation and calibration of a compact Ward square against a large-spin or rotor target. Deriving a native spin-2 phase from finite-spin dynamics is a further problem.

Manuscript · p. 1, abstract

Tower-Filtered ED Lanczos Scaling Audit for the Compact Ward-Square Spin-2 Box

Evidence · One-Hamiltonian composition requirements / Reduced finite-spin tower scaling

Finite-Spin / Rotor Ward-Square Implementation Audit

QATNU technical audit

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Abstract

We test the next microscopic route for the chiral spin-2 carrier: replacing the noncompact Ward-completed continuous-variable square by a compact rotor or large-spin symmetry sector. The audit is positive at the compact-rotor spin-wave and Holstein-Primakoff code-sector level. A particularly simple choice, $r_0 = 1$, realizes the synthetic factor with ordinary compact phase locking, has an exact diagonal $U(1)$ shift, gives one scalar Goldstone branch, therefore two transverse-traceless branches after tensoring with the TT quotient, and preserves the Ward stress response $\chi_S(0,k) \sim k^{-2}$. It avoids the large integer charge hierarchy of localized rational choices such as $r_0 = 1/2$, at the cost of spreading the finite-width synthetic mode across the width. The result is not yet a nonperturbative finite-d phase proof: compact vortices, finite-J anisotropies, and stability of the ordered phase remain the next checks.

1 Target

The effective continuous-variable construction has the Ward-completed square

$$H_m = \frac{\Omega^2}{2}\|S - \alpha h_{\mathrm{wall}}\|^2, \tag{1}$$

which ties the finite stress coupling to the seagull term and protects the pole by a diagonal shift. The finite-dimensional target is to realize the same structure with compact local degrees of freedom rather than exact canonical oscillators.

For a compact rotor, the noncompact factor $\theta_{s+1} - r\theta_s$ is replaced by an integer locking term

$$-J\cos(q\theta_{s+1} - p\theta_s), \qquad r = \frac{p}{q}. \tag{2}$$

The harmonic expansion reproduces the desired chiral factor after rescaling the coupling. The Ward square becomes the compact lock

$$-J_m\cos(\phi - \theta_0), \tag{3}$$

with diagonal shift vector $(1, r, r^2, \ldots; 1)$ in the harmonic theory.

2 Compactness tradeoff

Localized rational choices retain strong wall residue but require integer charges

$$Q_s \propto p^s q^{W-1-s} \tag{4}$$

for a compact global $U(1)$ action. This creates an exponential charge hierarchy unless $p = q = 1$. The audit therefore tests both localized and simple compact choices. The simple choice $r_0 = 1$ uses ordinary phase differences $\theta_{s+1} - \theta_s$ and a uniform diagonal shift. It is less wall-localized, with residue of order $1/W$, but for fixed synthetic width it still has finite residue and a finite second gap.

choice	r_0	p/q	z	α_χ	residue	$\Delta_2(0)$	doubler gap	$\log_{10} Q_{\max}$
legacy 21/50	0.42	21/50	0.999889	-1.999529	0.4516	0.5932	0.8999	25.5
localized 1/2	0.5	1/2	0.999867	-1.999531	0.4286	0.518	0.9075	4.52
moderate 2/3	0.6667	2/3	0.999815	-1.999537	0.3571	0.3675	0.9206	7.16
simple 1/1	1	1/1	0.999767	-1.999569	0.05882	0.1845	0.9393	0

Table 1: Compact rational synthetic factors at fixed width $W = 16$. All tested choices retain $z \simeq 1$ and $\chi_S \sim k^{-2}$. The localized choices have larger residue but pay a compact charge/harmonic-order cost.

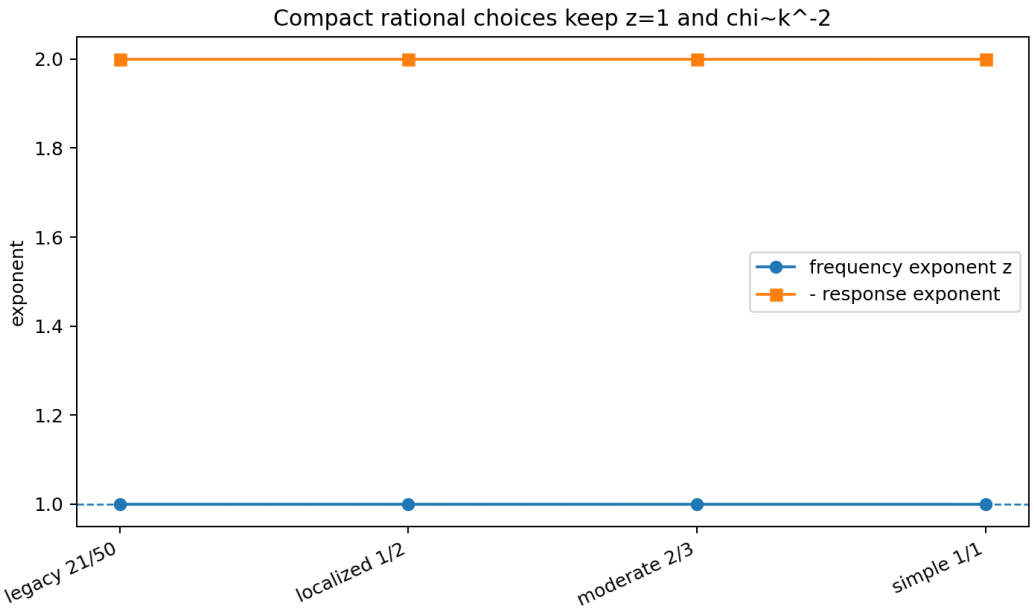

Figure 1: The protected branch and Ward response survive the compact rational choices.

3 Simple compact route: $r_0 = 1$

The most economical finite-dimensional candidate uses

$$D_s\theta = \theta_{s+1} - \theta_s, \tag{5}$$

with top Wilson pinning only away from the origin and the local Ward lock $\phi - \theta_0$. The diagonal symmetry is ordinary uniform phase rotation,

$$\theta_s \mapsto \theta_s + \epsilon, \qquad \phi \mapsto \phi + \epsilon. \tag{6}$$

The audit finds one scalar soft branch; after the TT quotient this gives exactly two positive TT branches.

4 Symmetry-preserving perturbations

Random positive perturbations built from the invariant rows

$$\theta_{s+1} - \theta_s, \qquad \phi - \theta_0 \tag{7}$$

annihilate the diagonal shift vector at $k = 0$. They preserve the massless pole and the Ward response. Generic local wall/stress terms that do not preserve the shift open a mass gap.

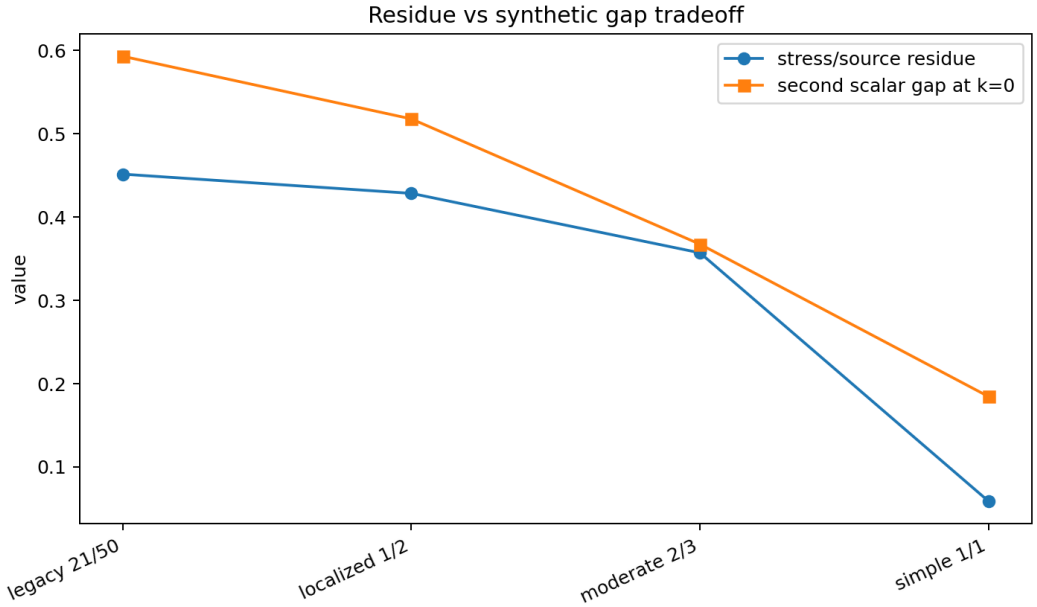

Figure 2: The simple compact choice has smaller but nonzero stress residue at fixed width.

protected perturbation samples	64/64 passed
z range	[0.999742637, 0.999803541]
χ exponent range	[-1.999579081, -1.999559149]
generic shift-breaking gap range	[0.0002275, 0.05793]

5 Large-spin code-sector estimates

The compact rotor can be realized in a finite-dimensional ordered spin sector through a Holstein-Primakoff expansion. The exact finite matrices need not obey $[q, p] = iI$; the massless mode is instead a thermodynamic Goldstone mode of the continuous $U(1)$ symmetry. The local Gaussian covariance remains finite in the audited boxes, giving controlled expectation-level Holstein-Primakoff errors.

J	local dimension	HP defect
16	33	5.959039e-03
32	65	2.979519e-03
64	129	1.489760e-03
128	257	7.448798e-04

Table 2: Large-spin ordered-sector estimates using the largest audited local occupation proxy.

6 Boundary of the result

This audit does not prove a complete nonperturbative finite-d graviton phase. It proves that the Ward-completed square has a compact finite-dimensional route at the spin-wave/ordered-sector level. Remaining checks are: (i) compact vortex or phase-slip relevance, (ii) stability of the ordered $U(1)$

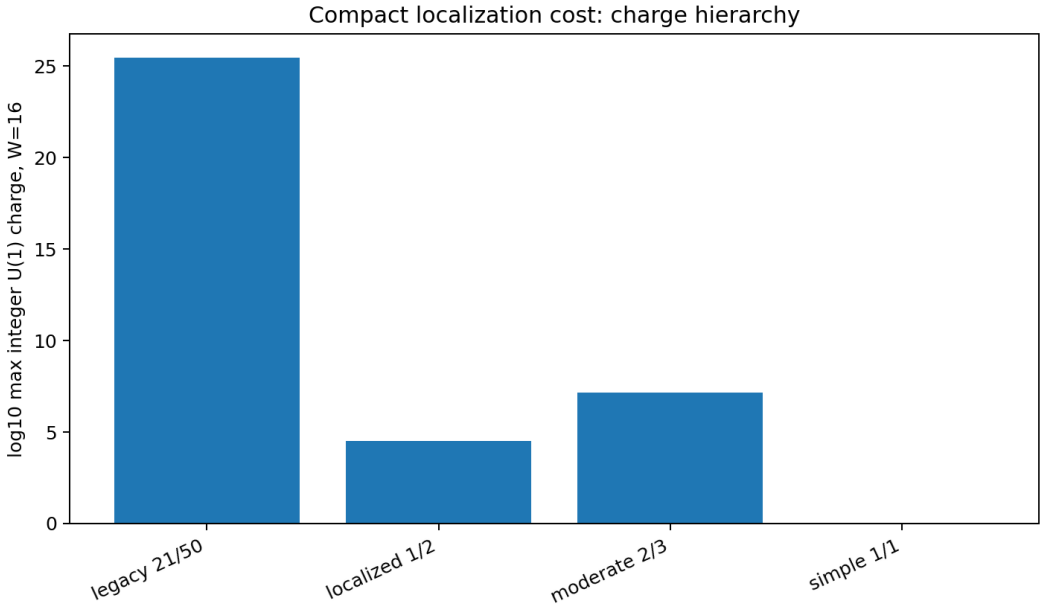

Figure 3: The price of compact localized locking is an integer charge hierarchy. The $r_0 = 1$ route avoids this.

phase in the full 3D lattice model, (iii) finite-J anisotropy and instanton effects, and (iv) retaining the first-class tensor quotient and absence of mirror branches in the full microscopic realization.

7 Conclusion

The next finite-dimensional route is viable but should be framed precisely. The best candidate is not an onsite oscillator truncation. It is a finite-spin or rotor Goldstone implementation of the Ward-completed covariant square. The simple $r_0 = 1$ compact variant avoids the severe charge hierarchy and passes the linear branch, response, doubler-gap, and protected-perturbation tests, at the cost of weaker wall localization. This identifies the next decisive calculation: a nonperturbative compact-spin simulation or analytic stability proof of this ordered Ward-square phase.

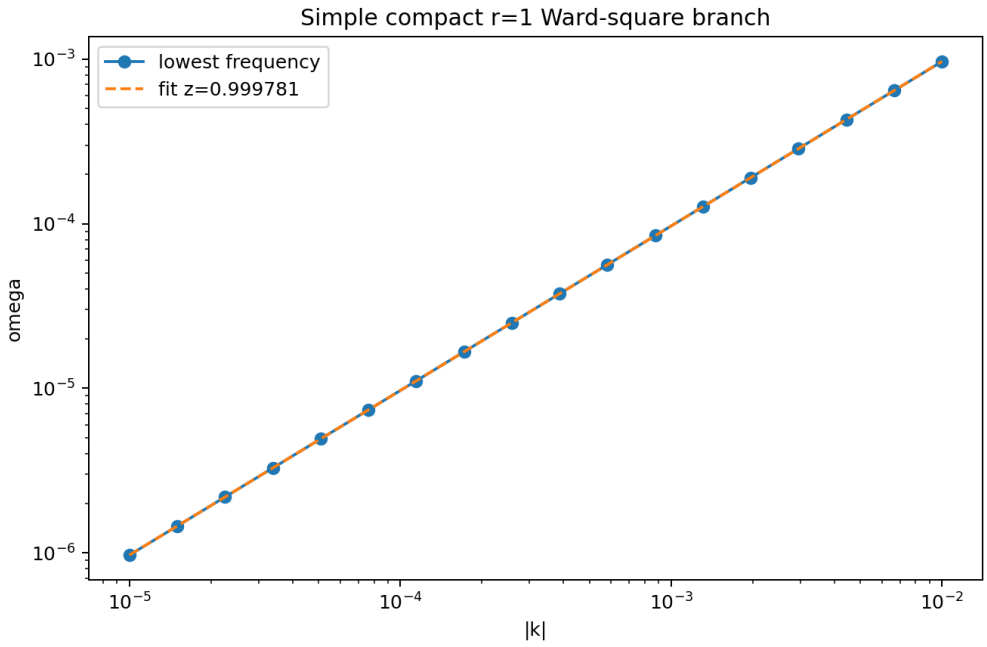

Figure 4: Frequency of the simple compact $r_0 = 1$ branch.

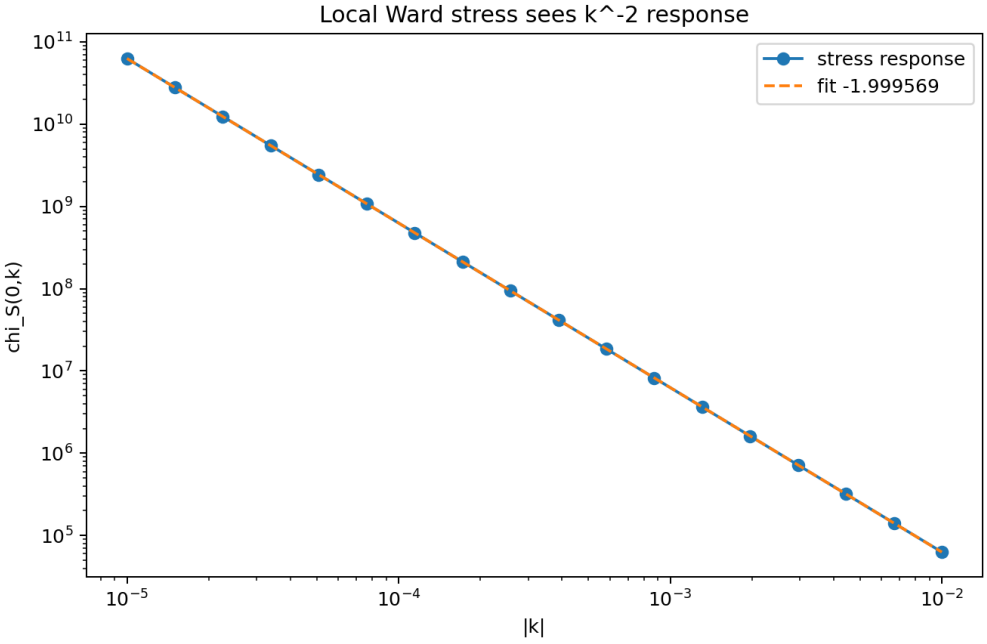

Figure 5: The local Ward stress variable has the expected static k^{-2} response.

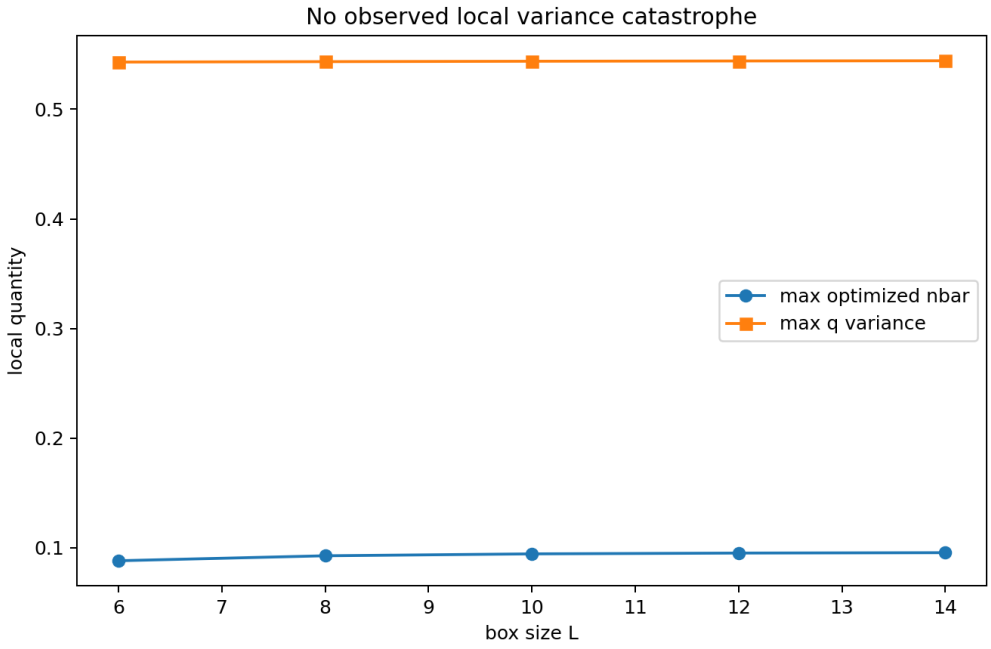

Figure 6: Local covariance diagnostics for the simple compact route. No local variance catastrophe appears in the audited sizes.

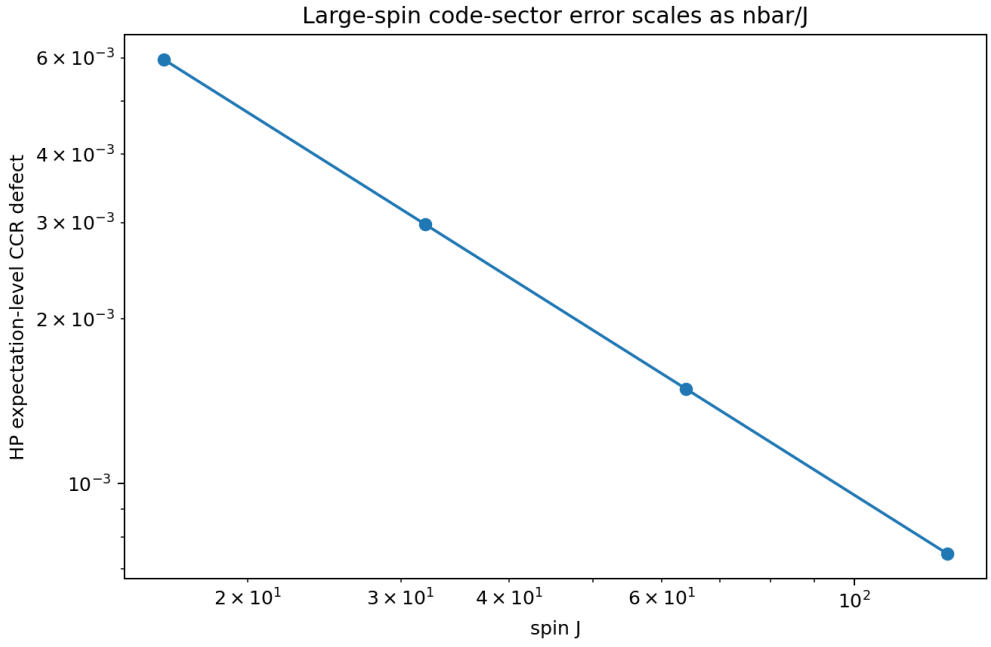

Figure 7: Expectation-level large-spin defect estimated as $\bar{n}/J$.