CORRECTION AND SCOPE

Finite-Spin Compact-Rotor Phase Screen

Original title · Finite-Spin Compact-Rotor Nonperturbative Phase Probe for the Ward-Square Route

The compact-rotor probe tests finite local dimensions in the specified sector. Its measured observables characterize that screen. A nonperturbative tensor phase remains unestablished.

Manuscript · p. 1, abstract

Tower-Filtered ED Lanczos Scaling Audit for the Compact Ward-Square Spin-2 Box

Evidence · Reduced finite-spin tower scaling / One-Hamiltonian composition requirements

Finite-Spin / Compact-Rotor Nonperturbative Phase Probe for the Ward-Square Route

QATNU technical audit

July 2, 2026

Abstract

We run the next finite-local-dimension screen for the $r_0 = 1$ compact Ward-square route. The previous audit established the desired spin-wave target: one compact diagonal Goldstone branch, two TT branches after quotient tensoring, $z = 1$, and $\chi_S(0, k) \sim k^{-2}$. Here we test the nonperturbative assumption behind that target with a three-dimensional compact diagonal $U(1)$ rotor surrogate. The result is positive as a phase screen: the low-temperature regime has finite magnetization and helicity stiffness with suppressed plaquette vorticity, while the high-temperature regime is disordered. However, this is still not a proof of the full finite-spin tensor/matter phase. Exact Ward Gauss-law implementation, finite-spin quantum dynamics, and exclusion of additional tensor-sector Goldstones remain separate obligations.

1 Scope

The surrogate model is

$$H_{XY} = -\sum_{\langle xy \rangle} \cos(\theta_x - \theta_y), \tag{1}$$

which isolates the compact diagonal $U(1)$ ordered sector required by the Ward-completed square. It intentionally omits the full synthetic-wall tensor quotient and the microscopic stress Gauss law. Its pass condition is therefore modest: it should show that the diagonal compact $U(1)$ symmetry can support an ordered Goldstone regime in three spatial dimensions without requiring a noncompact oscillator.

2 Monte Carlo screen

We used checkerboard Metropolis sweeps on periodic L^3 lattices with $L = 6, 8, 10, 12$. The sampled inverse temperatures were $\beta = 0.30, 0.45, 0.50, 0.65, 0.80, 1.20$. Measured diagnostics were the magnetization $|m|$, energy density, helicity modulus proxy Υ_x, and plaquette winding fraction. This is a screening run, not a precision critical-exponent computation.

3 Largest-size diagnostics

β	$\lvert m\rvert_{L=12}$	Υ_x	vortex fraction	screen
0.30	0.041833	-0.012428	0.256384	disordered-screen
0.45	0.326150	0.163176	0.155977	crossover-screen
0.50	0.420935	0.126674	0.100653	crossover-screen
0.65	0.751211	0.629673	0.012253	ordered-screen
0.80	0.816466	0.729784	0.002446	ordered-screen
1.20	0.889293	0.841305	0.000005	ordered-screen

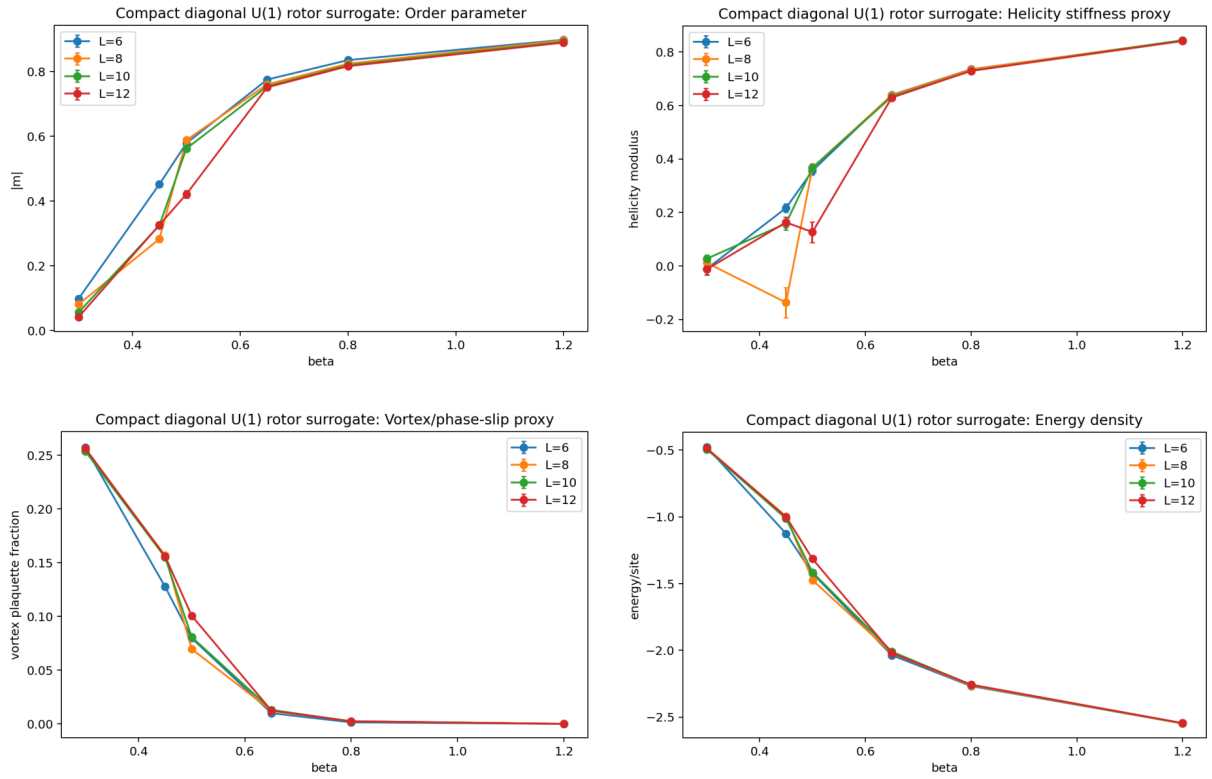

Figure 1: Compact diagonal $U(1)$ rotor surrogate. A low-temperature ordered regime has finite magnetization/helicity and suppressed plaquette winding.

The ordered side is visible by $\beta = 0.65$ and strong by $\beta = 0.80, 1.20$. This supports the compact-Goldstone assumption used by the $r_0 = 1$ Ward-square spin-wave construction.

4 Finite-spin code-sector scale

The spin-wave local density from the harmonic Ward-square audit was

$$\bar{n}_{\max,\infty} \simeq 0.100408161493. \tag{2}$$

Thus a Holstein-Primakoff finite-spin realization has ordered-sector defect scale $\bar{n}/J$:

J	local dimension $2J+1$	$\bar{n}/J$
16	33	6.275510e-03
32	65	3.137755e-03
64	129	1.568878e-03
128	257	7.844388e-04
256	513	3.922194e-04

This remains an ordered-sector estimate, not an operator-norm finite-code proof.

5 Anisotropy and exact symmetry

The audit also clarifies the role of exact diagonal $U(1)$. A clock anisotropy

$$-\epsilon \cos(q\theta) \tag{3}$$

is mass relevant; harmonically it generates

$$\Delta_q \sim q\sqrt{\epsilon}. \tag{4}$$

Therefore a finite-spin construction must preserve the diagonal Ward $U(1)$ exactly, or make symmetry breaking parametrically irrelevant by a separate mechanism. A finite local Hilbert space is compatible with the route; explicit finite clock anisotropy is not.

6 Verdict

The result is positive as a nonperturbative compact-rotor phase screen: the simple $r_0 = 1$ route has a plausible finite-spin ordered phase behind its harmonic $z = 1$, k^{-2} response. But the full QATNU spin-2 claim is still not closed. The remaining technical obligations are: construct the microscopic finite-spin Ward stress Gauss law, verify the full first-class tensor quotient in that finite-spin Hilbert space, and check that no extra Goldstone branches, mirror cones, or vortex/anisotropy effects spoil the two-TT sector.