Finite-Spin Flux-Saturation Audit for the Compact Ward-Square Tensor Chain

QATNU numerical audit

July 3, 2026

Verdict

This audit checks the finite-spin saturation obligation for the full compact Ward-square tensor chain in the ordered spin-wave sector. After removing the global diagonal Goldstone oscillator, the local covariance remains bounded with increasing box size. Finite spin J then controls the Holstein-Primakoff defect as $\bar{n}/J$ and gives exponentially small local saturation tails.

This is not a nonperturbative finite-d phase proof. It is a quantitative bounded-flux screen for the ordered sector that complements the compact Monte Carlo phase screens.

Model

The scalar part of the constrained tensor chain has one Ward stress wall and $W = 16$ tensor layers. The $r_0 = 1$ compact path stiffness is

$$K(k) = L_{\mathrm{path}} + |d(k)|^2 P_{\mathrm{tensor}},$$

where the spatial stiffness acts on the tensor layers. The trace and divergence constraints reduce each nonzero momentum to two TT directions. The covariance calculation is therefore carried out for one scalar TT copy and then interpreted with two TT polarizations.

Local covariance

For a harmonic oscillator mode with frequency $\omega_m(k)$ and eigenvector u_m, the local variances are

$$\langle q_\ell^2 \rangle = \frac{1}{V} \sum_{k,m} \frac{|u_m(\ell, k)|^2}{2\omega_m(k)}, \qquad \langle p_\ell^2 \rangle = \frac{1}{V} \sum_{k,m} \frac{\omega_m(k)|u_m(\ell, k)|^2}{2},$$

with the single global zero oscillator removed. The optimized one-site occupation proxy is

$$\bar{n}_\ell = \sqrt{\langle q_\ell^2 \rangle \langle p_\ell^2 \rangle} - \frac{1}{2}.$$

L	max $\langle q^2 \rangle$	max $\langle p^2 \rangle$	max $\bar{n}$	stress $\bar{n}$
6	0.539430	1.388757	0.021322	0.008977
8	0.539151	1.388807	0.020849	0.008866
10	0.539109	1.388818	0.020769	0.008851
12	0.539111	1.388821	0.020765	0.008854
16	0.539132	1.388823	0.020788	0.008865
20	0.539148	1.388823	0.020807	0.008873
24	0.539158	1.388824	0.020820	0.008877

For the largest audited box, $L = 24$,

$$\max_\ell \langle q_\ell^2 \rangle = 0.539157941, \quad \max_\ell \langle p_\ell^2 \rangle = 1.388823655, \quad \max_\ell \bar{n}_\ell = 0.020820302.$$

The extrapolated infinite-volume value is

$$\max_\ell \bar{n}_\ell \to 0.020757175.$$

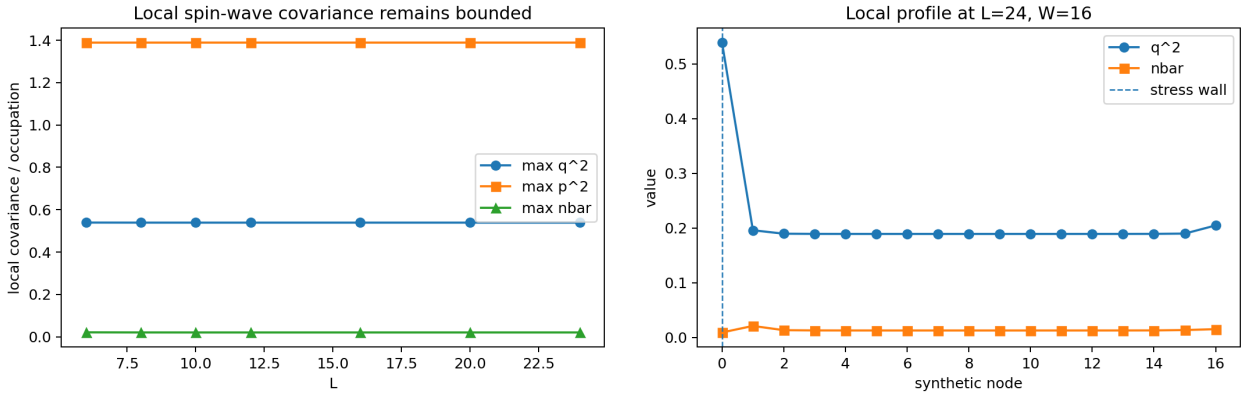

Figure 1: Local covariance scaling and synthetic-node profile.

Finite-spin saturation laws

Using a geometric tail proxy with mean $\bar{n}_{\max}$, the local tail is

$$P(N > J) \simeq \eta^{J+1}, \qquad \eta = \frac{\bar{n}}{1+\bar{n}}.$$

For two TT polarizations on $(W+1)L^3$ local sites, the expected number of saturated sites is $2(W+1)L^3\eta^{J+1}$.

J	local dim.	HP defect $\bar{n}/J$	local tail	expected saturated sites
8	17	2.603e-03	6.107e-16	2.870e-10
16	33	1.301e-03	1.829e-29	8.595e-24
32	65	6.506e-04	1.640e-56	7.707e-51
64	129	3.253e-04	1.318e-110	6.195e-105
128	257	1.627e-04	8.519e-219	4.004e-213
256	513	8.133e-05	0.000e+00	0.000e+00

Meaning

The finite-spin route is not blocked by a local spin-wave variance catastrophe. In the ordered sector, moderate spin already makes the Holstein-Primakoff correction small, and larger spin suppresses saturation rapidly. This is qualitatively different from trying to impose exact finite-dimensional canonical commutators at a site: the massless mode is collective and protected by the diagonal Ward symmetry.

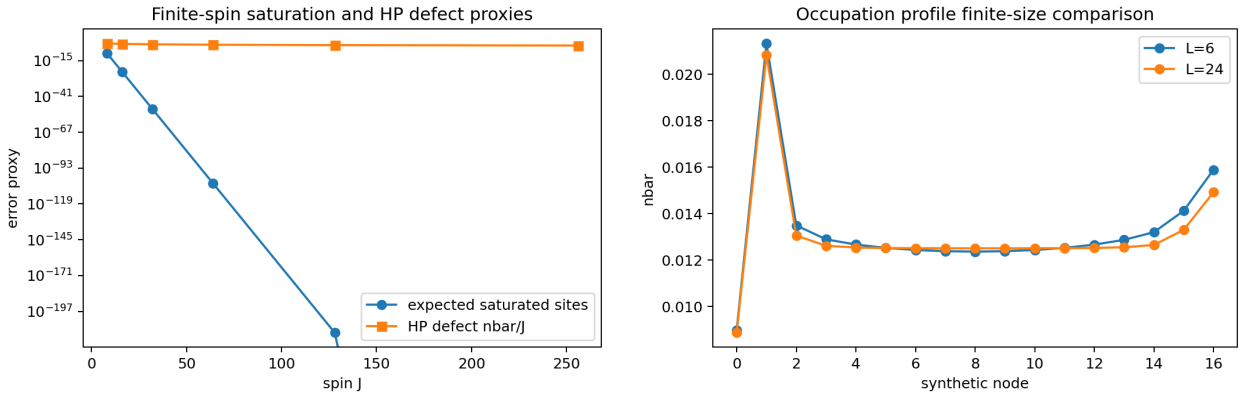

Figure 2: Finite-spin saturation proxies and occupation profiles.

Boundary

The audit is Gaussian and ordered-sector. It does not prove deconfinement, compact defect irrelevance, exact hard Gauss-law dynamics, or the full many-body spectral pole. It narrows the next tests to direct compact finite-spin dynamics and hard-constraint/deconfined simulations.