External Coupling Kernels for Internal Scalar Loads

Purpose

The four-paper scalar-selection program now has a clear internal scalar load Λ_i defined on a medium graph. The remaining conceptual gap is the bridge from this internal load to an external probe. This note classifies possible internal-to-external coupling kernels and records what must be measured to distinguish them.

1 Basic Map

Paper 4 derives an internal scalar load field on the graph vertices,

$$(\kappa L_w + \mu^2 I)\Lambda = A\rho. \tag{1}$$

An external sensor at position x does not, in general, measure Λ_i directly. The minimal bridge is a kernel map

$$\Lambda_{\text{ext}}(x) = \sum_i C(x,i)\Lambda_i. \tag{2}$$

The nature of $C(x,i)$ determines whether the scalar sector is externally accessible, how it decays with distance, and what experimental geometry is appropriate.

2 Kernel Classes

2.1 Near-field / evanescent kernel

The external response is dominated by short-range coupling,

$$C_{\text{ev}}(x,i) \sim C_0\, e^{-d(x,i)/\ell_c}, \tag{3}$$

where $d(x,i)$ is physical distance from the external probe to vertex i and ℓ_c is a coupling depth. Observable effects are confined to small distances. This class predicts a strong dependence on probe standoff and substrate thickness.

2.2 Substrate-mediated diffusive kernel

The scalar load couples into another medium degree of freedom (mechanical, thermal, dielectric, or phononic) which then carries the signal outward. A minimal coarse-grained form is

$$(\eta\nabla^2 - m_s^2)\Phi_s(x) = B\sum_i \delta(x - x_i)\Lambda_i, \tag{4}$$

with measured response $\Lambda_{\text{ext}}(x) = \Phi_s(x)$. This class predicts finite propagation delays and potentially broad relaxation tails.

2.3 Radiative / boundary-modulated kernel

The internal scalar load modulates a boundary condition or effective impedance seen by an electromagnetic or microwave mode. In the linearized regime one may write

$$\Lambda_{\mathrm{ext}}(x,\omega) = \sum_i C_{\mathrm{rad}}(x,i;\omega)\Lambda_i, \tag{5}$$

where C_{rad} is frequency dependent. This class predicts resonant enhancement and strong sensitivity to cavity or resonator tuning.

2.4 Vacuum / effective Green-response kernel

The most ambitious possibility is that the internal scalar load sources an effective external scalar field directly through a spatial Green kernel,

$$\Lambda_{\mathrm{ext}}(x) = \sum_i G_{\mathrm{ext}}(x,x_i)\Lambda_i, \tag{6}$$

with

$$G_{\mathrm{ext}}(r) \sim \begin{cases} r^{2-d_{\mathrm{eff}}^{\mathrm{ext}}}, & d_{\mathrm{eff}}^{\mathrm{ext}} \neq 2, \\ \log r, & d_{\mathrm{eff}}^{\mathrm{ext}} = 2. \end{cases} \tag{7}$$

This is the strongest externalization claim and requires the clearest evidence.

2.5 Null kernel

A perfectly respectable outcome is

$$C(x,i) \approx 0 \tag{8}$$

outside the medium. In that case the scalar load is an internal medium observable only. This does not kill the program, but it limits what can be claimed experimentally.

3 Observable Consequences

Different kernel classes imply different signatures.

Kernel class	Distance law	Distinctive signature
Near-field / evanescent	exponential / short-range	Strong suppression with probe standoff, weak large-distance tail
Substrate-mediated diffusive	medium-dependent	Time-delayed and broadened response, dependence on substrate properties
Radiative / boundary-modulated	resonant / mode-dependent	Frequency-selective enhancement and phase-sensitive readout
Vacuum / effective Green-response	power law or screened power law	Spatial law not reducible to simple substrate coupling
Null	none	Internal sensor sees effect; external sensor sees nothing

4 How to Distinguish Them

A minimal external-coupling experiment should vary:

1. probe distance from the medium,
2. substrate thickness and material,
3. drive frequency / switching rate,
4. graph topology at fixed mass,
5. shielding and cavity boundary conditions.

The goal is not to identify the kernel in one shot, but to eliminate classes. For example, if the response vanishes rapidly with standoff and changes strongly with substrate, the evanescent or substrate-mediated classes dominate. If the response follows cavity tuning, a radiative kernel is implicated. If a clean power-law tail survives controls, the vacuum-like class becomes more plausible.

5 Minimal Coupling Note for the Program

The strongest statement currently justified by the four-paper arc is:

> The existence of an internal scalar load does not by itself imply an externally measurable scalar geometry. An explicit coupling kernel $C(x, i)$ must be specified, measured, or bounded.

This note isolates that requirement so the distinction between internal medium observables and external geometric observables remains sharp.

6 Next Measurement Priorities

1. Bound the null case: determine whether any external sensor sees a reproducible signal at all.
2. Fit distance-law data to screened-exponential versus power-law models.
3. Test whether temporal signatures track the graph Laplacian spectrum or ordinary thermal/substrate response.
4. Determine whether topology-switching at fixed mass changes the external signal more strongly than purely geometric relocation of mass.