QATNU covariant Ward-square / doubly-constrained seagull audit

Verdict: positive at the harmonic/local-constraint level. The previous obstruction was caused by trying to write an order-one h^2 seagull directly as a bounded-range tensor square. The enlarged local formulation instead introduces a covariant difference R = S - alpha A and imposes local Ward constraints div R = 0 and tr R = 0. Eliminating those local constraints generates the exact TT projector; it is not inserted as a nonlocal microscopic kernel. This is still not a nonperturbative finite-d phase proof.

Core local object:

Variables X=(A_ij,S_ij). Define R_ij=S_ij-alpha A_ij and H_square = (Omega^2/2) ||R||^2, with local constraints div R = 0 and tr R = 0. The combined gauge/trace shift delta A=g, delta S=alpha g leaves R, the square, and the constraints invariant.

Key numerical checks:

check	value
constraint nullspace equals TT projector	5.413e-12
div residual on nullspace	2.584e-16
trace residual on nullspace	6.384e-16
expanded square null residual	0.000e+00
constraint invariance residual	9.465e-17
rank formulas match	True
z scalar	0.999999803
chi_S exponent	-1.999989379
stress zero-mode weight	0.058823529
second gap at origin	0.184536719
low TT branches at |k|=1e-3	2
protected perturbations	64/64
detuning gap exponent	0.499890

Finite-lattice rank checks:

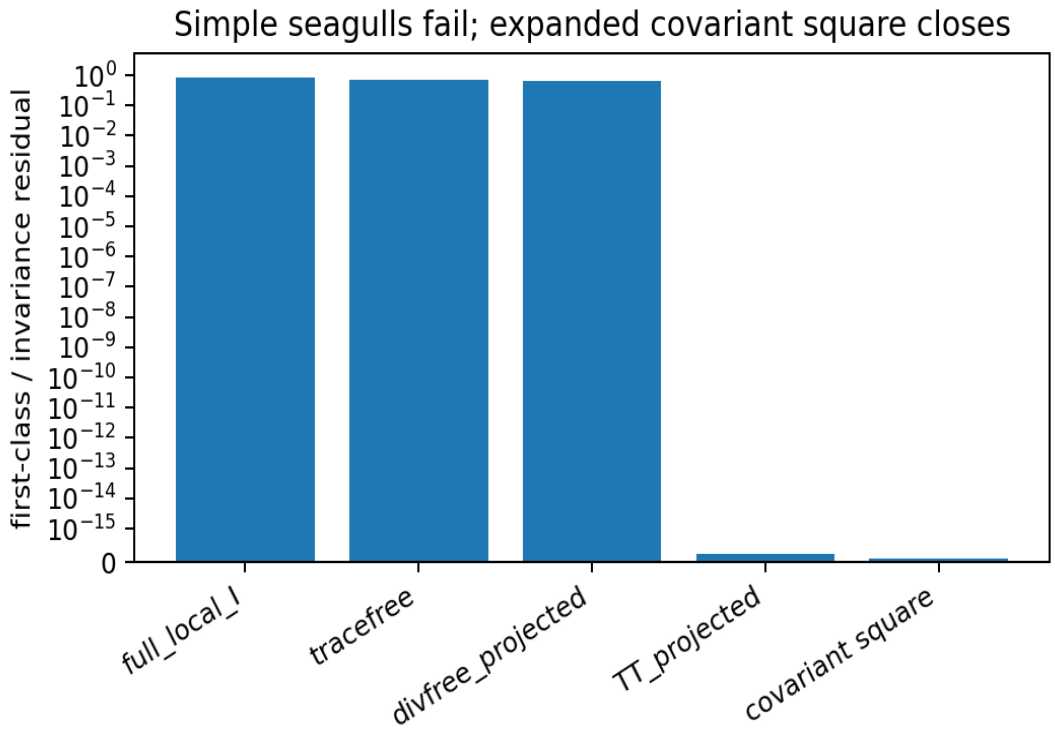
L	V	rank(div)	3(V-1)	rank(div,tr)	4V-3	dof	2V+3
2	8	21	21	29	29	19	19
3	27	78	78	105	105	57	57
4	64	189	189	253	253	131	131

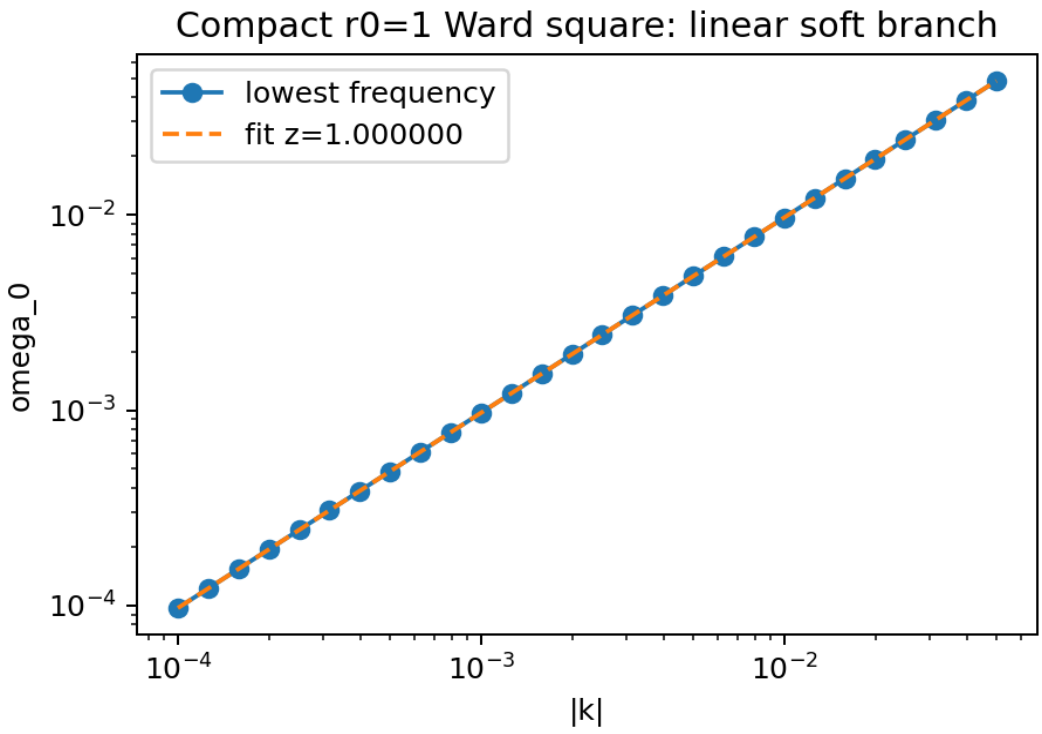
Interpretation:

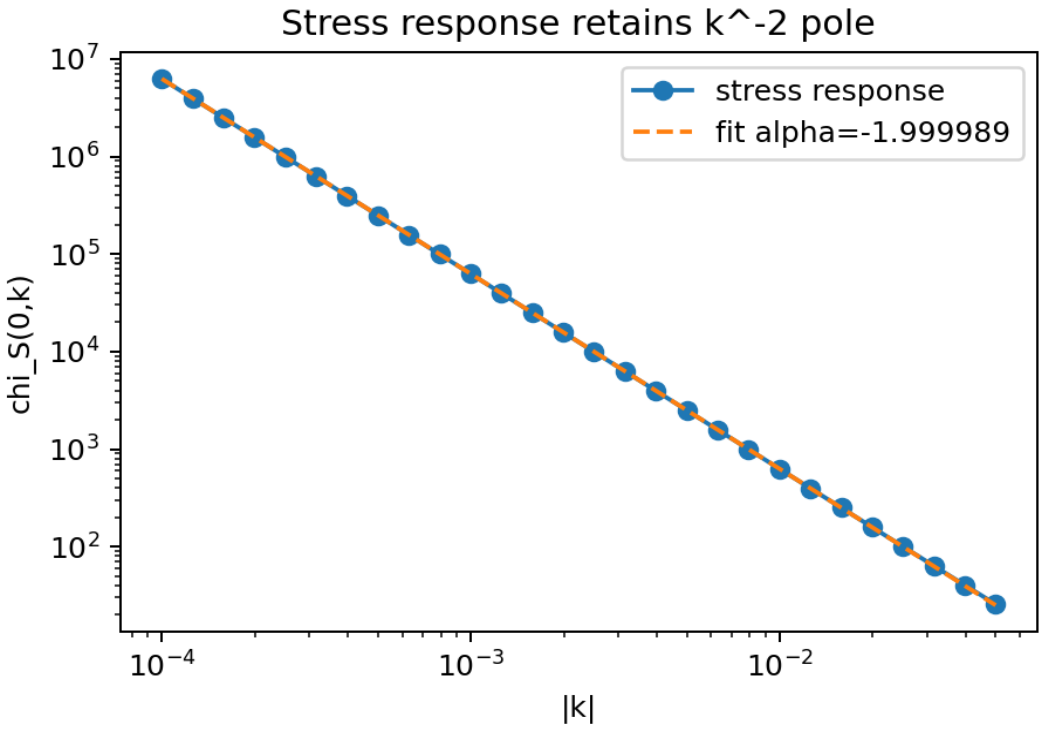
The local constraints div R=0 and tr R=0 have exactly the finite-lattice rank expected of local Gauss laws. At nonzero momentum their nullspace is the two-dimensional TT subspace. Thus the exact order-one TT seagull can be obtained from a local constrained covariant square in an enlarged field space. Written after eliminating constraints, the same operation looks like the algebraically nonlocal TT projector; before elimination it is local.

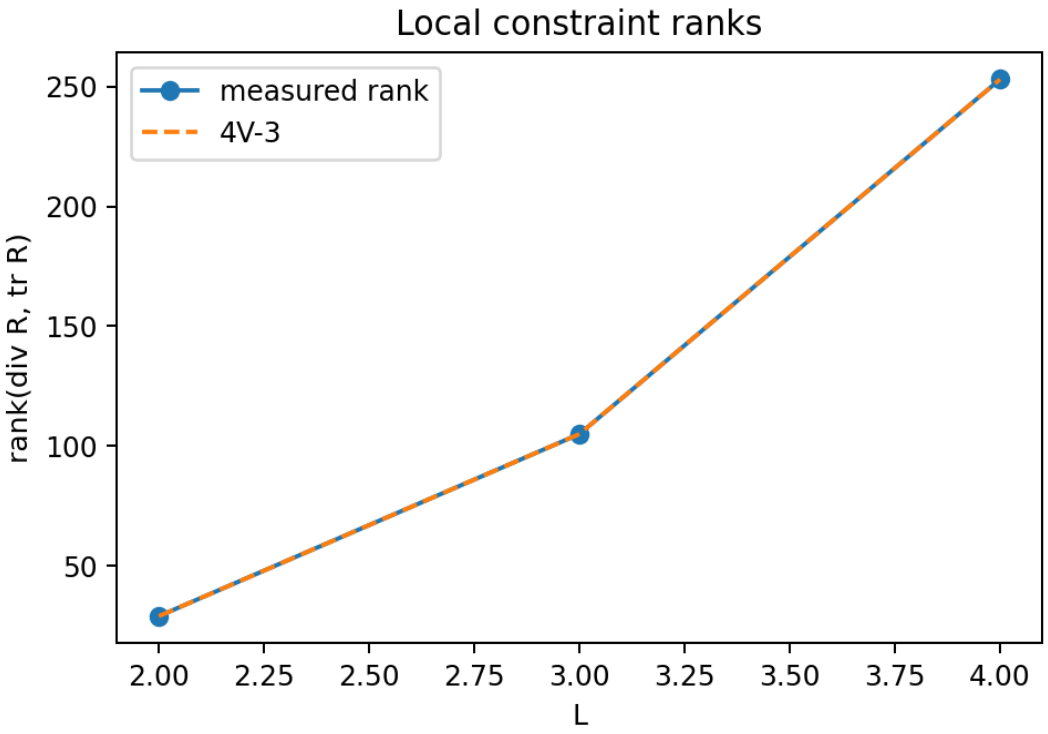
Boundary:

This audit is linear/harmonic and constraint-rank level. It does not prove the compact finite-spin phase, does not simulate vortices of the full tensor/matter model, and does not prove bounded-flux saturation is harmless. It does remove the specific local-seagull objection at the expanded gauge-system level.




Compact r0=1 Ward square: linear soft branch
lowest frequency
fit z=1.000000
omega_0
10^{-2}
10^{-3}
10^{-4}
10^{-4}
10^{-3}
10^{-2}
|k|


Stress response retains k^-2 pole
stress response
fit alpha=-1.999989
chi_S(0,k)
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{-4}
10^{-3}
10^{-2}
|k|


Local constraint ranks
measured rank
4V-3
rank(div R, tr R)
250
200
150
100
50
2.00
2.25
2.50
2.75
3.00
3.25
3.50
3.75
4.00
L