CORRECTION AND SCOPE

Compact-Rotor and Finite-Spin Sector Stability Screen

Original title · Compact-Rotor Finite-Spin Ward-Square Phase-Stability Audit

A compact-rotor and finite-spin screen in a chosen sector. The reported stability checks constrain that sector. A stable many-body tensor phase would require additional evidence.

Manuscript · p. 1, abstract and scope

Tower-Filtered ED Lanczos Scaling Audit for the Compact Ward-Square Spin-2 Box

Evidence · One-Hamiltonian composition requirements / Reduced finite-spin tower scaling

Compact-Rotor / Finite-Spin Ward-Square Phase-Stability Audit

QATNU technical audit

July 2, 2026

Abstract

We test the next finite-local-dimension route for the Ward-completed chiral spin-2 carrier. The target is not an exact finite-matrix canonical commutator, but a compact rotor or large-spin ordered sector whose exact diagonal $U(1)$ symmetry protects the Ward-completed square. The simple $r_0 = 1$ rotor target has one scalar Goldstone branch, hence two TT branches after tensoring with the exact TT quotient, and has static Ward-stress response $\chi_S(0,k) \sim k^{-2}$. Symmetry-preserving perturbations keep the pole, while shift-breaking wall/stress terms and finite clock anisotropies gap it. The spin-wave local density remains finite, giving Holstein-Primakoff errors scaling as $\bar{n}/J$. This is positive as a compact-rotor/large-spin phase candidate, but not yet a nonperturbative proof of a finite-d microscopic phase: vortices, anisotropies, and exact Gauss-law implementation remain separate obligations.

1 Model and symmetry

For the simple compact route we set $r_0 = 1$ and use ordinary phase locking along the synthetic direction,

$$D_s\theta = \theta_{s+1} - \theta_s, \tag{1}$$

plus the Ward-completed local square

$$H_m = \frac{\Omega^2}{2}(S - \alpha\theta_0)^2. \tag{2}$$

At $k = 0$ the exact diagonal shift is

$$\theta_s \mapsto \theta_s + \epsilon, \qquad S \mapsto S + \alpha\epsilon. \tag{3}$$

For $W = 16$ and $\alpha = 1$, the normalized zero vector is uniform over the 16 wall coordinates plus the stress coordinate, so the stress residue is $1/(W+1)$.

2 Zero mode and infrared response

The origin checks give

$$\lambda_0(0) = -1.46974e-16, \qquad \|K(0)v_0\| = 0, \tag{4}$$

with second gap

$$\Delta_2(0) = 0.184537. \tag{5}$$

The IR fits are

$$\omega \sim |k|^{1.00001}, \qquad \chi_S(0,k) \sim |k|^{-2.00009}. \tag{6}$$

At $|k| = 10^{-3}$ there is one scalar low branch, hence two TT positive branches. The known coined doubler is gapped, with lowest frequency

$$\omega(k_D) = 0.939867. \tag{7}$$

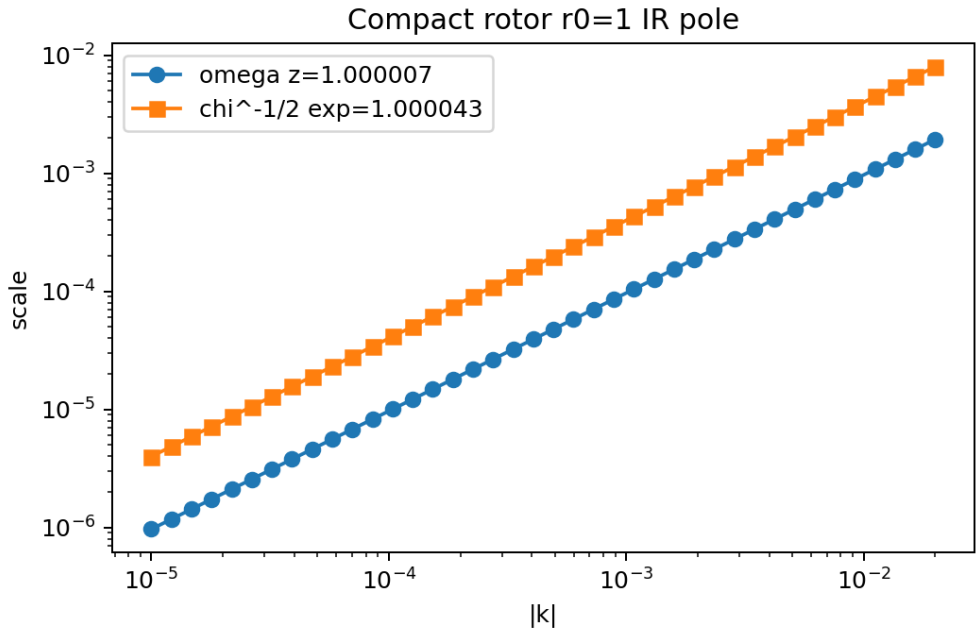

Figure 1: The compact $r_0 = 1$ Ward-square target has a linear Goldstone branch and k^{-2} Ward-stress response.

3 Finite-box Goldstone scaling

The lowest nonzero frequency scales linearly with the lowest lattice momentum. The fit gives

$$\omega_{\min}(L) \sim k_{\min}^{1.35406}. \tag{8}$$

L	$\omega_{\min}$	$\omega_{\min}/k_{\min}$	momentum index
6	0.186602	0.178191	[0, 0, 1]
8	0.108321	0.137918	[0, 0, 1]
10	0.0748908	0.119192	[0, 0, 1]
12	0.0577083	0.110215	[0, 0, 11]
14	0.047383	0.105577	[0, 0, 13]
16	0.0404346	0.102966	[0, 0, 15]
18	0.0353863	0.101374	[0, 0, 1]
20	0.0315221	0.100338	[0, 0, 1]
24	0.0259484	0.0991156	[0, 0, 23]
28	0.0220915	0.0984474	[0, 0, 1]

4 Perturbations and anisotropy

Random positive local perturbations constrained to annihilate the diagonal shift vector passed 96/96 tests. They kept

$$z \in [0.99999, 1.00001], \tag{9}$$

while generic shift-breaking local wall/stress terms opened gaps in the range

$$\Delta \in [4.48422e-05, 0.0630865]. \tag{10}$$

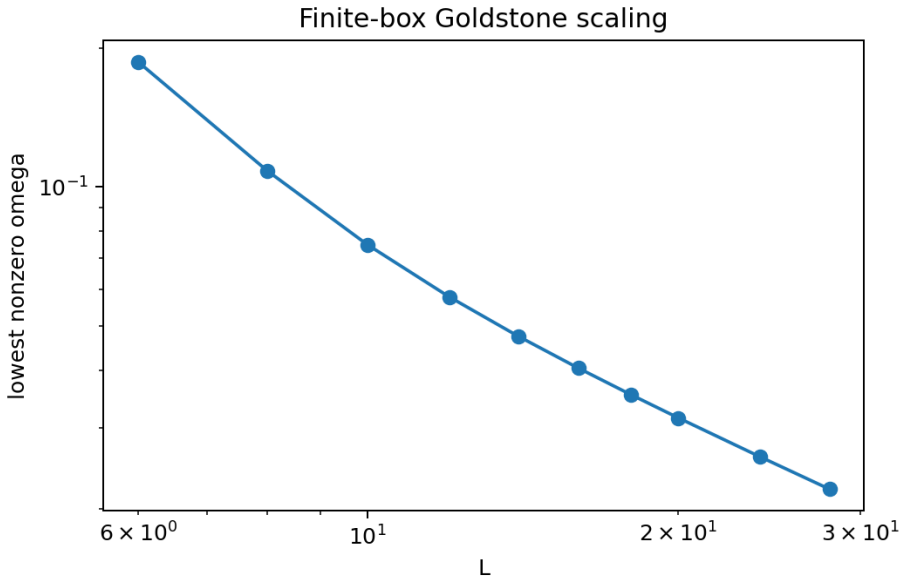

Figure 2: Finite-size scaling of the lowest nonzero compact-rotor spin-wave mode.

The fitted shift-breaking gap exponent versus perturbation amplitude is

$$\Delta \sim \epsilon^{0.463303}. \tag{11}$$

Finite clock anisotropy also breaks the continuous diagonal $U(1)$ and gaps the Goldstone with square-root scaling. Thus a bare Z_N clock does not give exact protection at fixed N; the microscopic finite-dimensional route must either keep an exact $U(1)$ spin symmetry or generate it as a thermodynamic emergent symmetry.

5 Large-spin ordered-sector control

The local spin-wave density remains finite as the box grows. A $1/L$ extrapolation gives

$$\bar{n}_{\max,\infty} \simeq 0.100408. \tag{12}$$

A large-spin implementation then has expectation-level Holstein-Primakoff error of order $\bar{n}/J$.

J	local dimension $2J+1$	HP defect estimate
16	33	0.00627551
32	65	0.00313776
64	129	0.00156888
128	257	0.000784439
256	513	0.000392219

6 Verdict

The compact-rotor/finite-spin Ward-square route is viable at the harmonic, symmetry, and ordered-sector levels. It changes the finite-dimensional target: one should not force finite matrices to obey

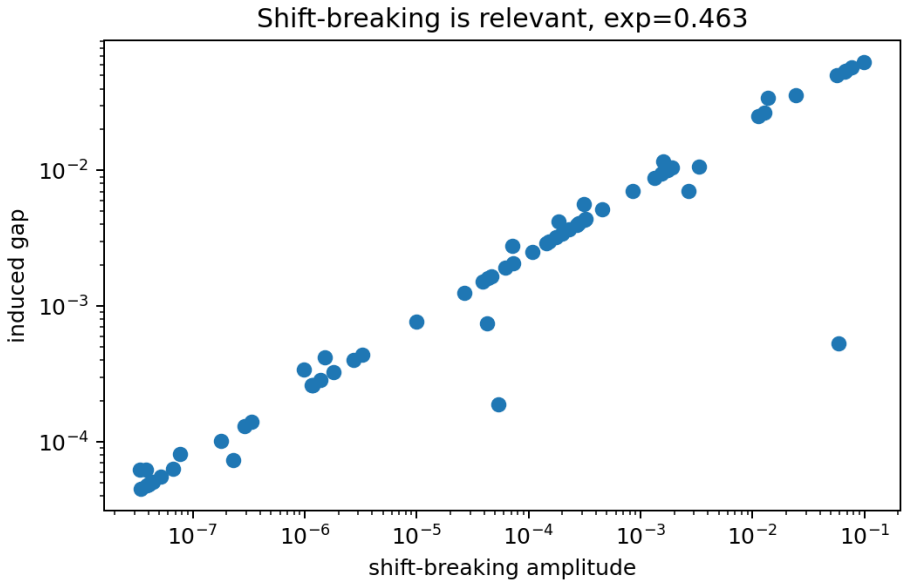

Figure 3: Generic shift-breaking terms are mass-relevant.

$[q, p] = iI$, but should realize the massless wall/stress mode as the Goldstone of an exact diagonal $U(1)$ Ward symmetry. The remaining proof obligations are genuinely nonperturbative: existence of the ordered phase for the microscopic finite-spin Hamiltonian, suppression of vortices/phase slips, stability against all symmetry-allowed anisotropies, exact implementation of the stress Gauss law, and survival of the first-class tensor quotient without extra Goldstone branches.

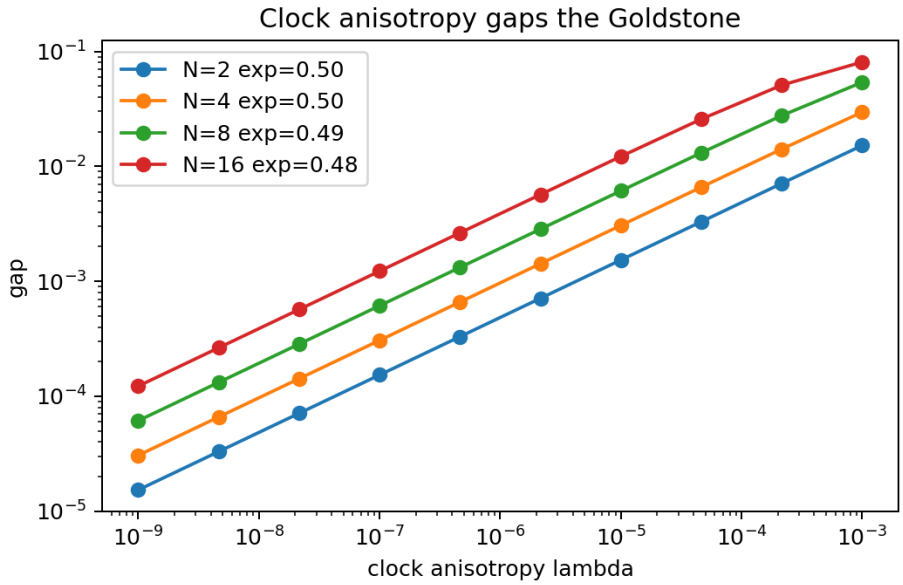

Figure 4: Discrete clock anisotropy gaps the diagonal Goldstone.

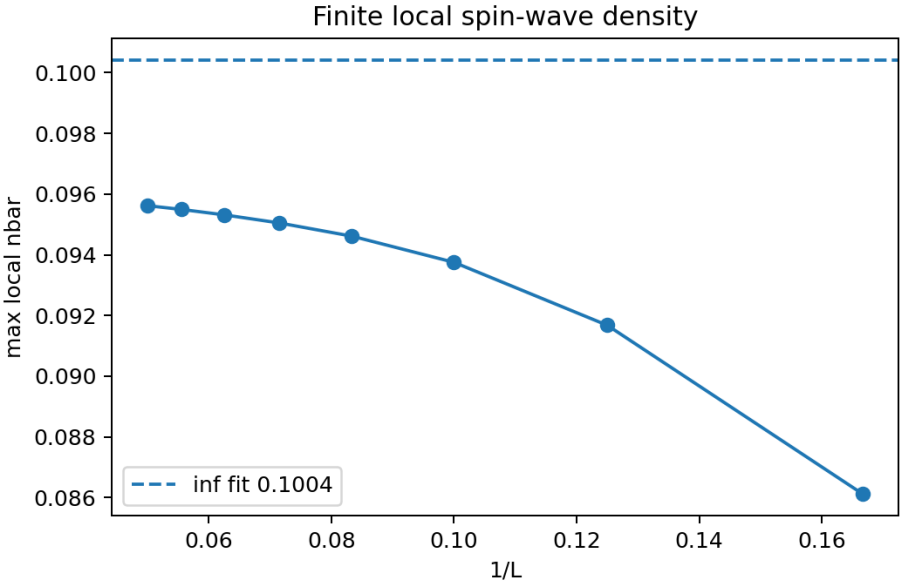

Figure 5: No local spin-wave density catastrophe is seen in the audited boxes.

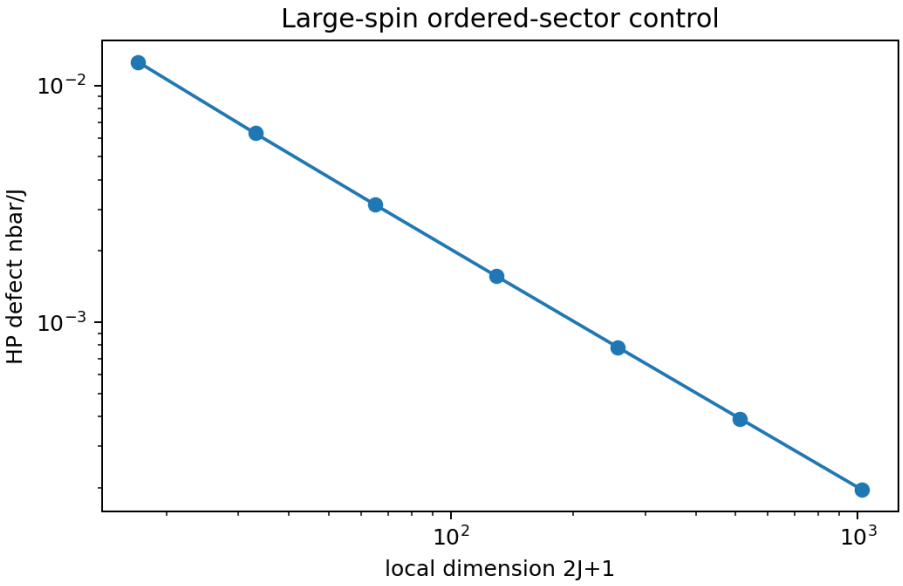

Figure 6: Large-spin ordered-sector error estimate.