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One Obstruction, Three Faces: A Local First-Class Chiral Bosonic Spin-2 Carrier and the Barrier to a Protected Graviton Phase

Earlier three-part formulation of the chiral carrier obstruction. The four-part edition expands the development within its stated model class.

Manuscript · p. 1, three-face formulation

One Obstruction, Four Faces: The Barrier to an Emergent Linear Graviton on a Local Lattice

Evidence · One-Hamiltonian composition requirements

One Obstruction, Three Faces: A Local First-Class Chiral Bosonic Spin-2 Carrier and the Barrier to a Protected Graviton Phase

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Abstract

We assemble and audit a local, first-class, chiral bosonic carrier for transverse-traceless (TT) spin-2 modes, and we use it to isolate exactly what separates a working linear spin-2 *carrier* from a protected emergent *graviton phase*. The carrier is positive by construction, has exactly two TT branches, linear dispersion $z = 1$, a finite boundary residue, and—when probed by an externally conserved TT source—the expected static $1/k^2$ susceptibility with TT Ward identities at machine precision. We then show that three apparently distinct obstacles to upgrading this object to a graviton are a single algebraic fact wearing three masks: the absence of a low-derivative-order local gauge-invariant spin-2 operator. That one fact (i) forces Lifshitz scaling $z = 3$ for any exact compact finite-dimensional local-character realization; (ii) Cotton-suppresses the response to a strictly local TT source to $O(k^4)$; and (iii) leaves the masslessness unprotected, since a positive local first-class mass is relevant. The chiral continuous-variable carrier evades the first face by abandoning finite local dimension and removing the lattice doubler, but faces (ii) and (iii) survive because they are the same fact. We state precisely what a genuine emergent graviton phase must add: an external (matter) Ward identity or symmetry that forbids the relevant mass and supplies a conserved source. All quantitative claims below were reproduced from independent re-implementations of the audited models.

1 Claim boundary

This paper does not claim emergent gravity. It claims, and audits, a *linear spin-2 response carrier*: a local, positive, first-class architecture whose constrained infrared sector contains exactly two TT branches with

$$\omega(k) = c_g\,|k| + O(|k|^3), \qquad N_+(k) = 2, \qquad Z_{\mathrm{local}}(k \to 0) > 0, \tag{1}$$

together with the correct conserved-source susceptibility. It then maps, sharply and negatively, the distance from that carrier to a symmetry-protected graviton. The negative half is the contribution we most want to be believed: the program has, by its own auditing, located a single obstruction and shown that the natural constructions move it rather than remove it.

2 The unifying fact

Write a local spin-2 operator on a translation-invariant lattice as a polynomial in the lattice momentum k. The physically meaningful operators are the gauge-invariant ones: those that survive the linearized diffeomorphism (Stueckelberg) quotient and are transverse-traceless. The organizing fact is a statement about the *minimal polynomial degree* of such operators.

Observation 1 (No low-order local gauge-invariant TT operator)**.** *In the linearized 3D tensor module, there is no nonzero local TT creation operator through derivative order two. The first exact local TT module enters at Cotton order (third derivative order). Equivalently, the minimal gauge-invariant TT characters are high degree in k: the transverse projector itself is degree zero but nonlocal, while every* local *representative is pushed to Cotton order.*

Observation 1 is not new physics; the Cotton tensor is the standard 3D conformal curvature, and its third-derivative order is classical. What is new is the recognition that the three obstructions audited below are all consequences of it. We first record what the carrier achieves (Section 3), then exhibit the three faces (Section 4).

3 The chiral bosonic carrier

3.1 First-class Stueckelberg quotient

The physical variables are symmetric tensors A_{ij} (coordinates) and B_{ij} (momenta) with one Stueckelberg pair (ϕ, π): fourteen real dimensions per momentum. For a local connection vector $n(k)$ define the five constraints

$$T = \operatorname{tr} A + \phi, \qquad D = n_i n_j A_{ij}, \qquad L_j = n_i B_{ij} - n_j \operatorname{tr} B + 2 n_j \pi. \tag{2}$$

Proposition 1 (Abelian first class)**.** *For every nonzero n, the five constraints* (2) *are first class: all ten Poisson brackets vanish identically. The quotient reduces the fourteen-dimensional phase space to four physical dimensions, i.e. exactly two TT canonical pairs, removing the helicity-*0 *and helicity-*$\pm$1 *sectors.*

Audit. Direct evaluation of the constraint algebra gives a maximum bracket of 2.8×10^{-17} at a generic n and 3.6×10^{-15} worst over 2000 random connections; the trace–momentum bracket from $\operatorname{tr} A$ is cancelled by the Stueckelberg bracket $2 n_j \pi$, and the longitudinal D–L_j bracket cancels between $n_i B_{ij}$ and $-n_j \operatorname{tr} B$. The lifted coined map is symplectic to $\sim 10^{-15}$ and the reduced quasienergies are exactly $(-4\omega, -4\omega, +4\omega, +4\omega)$ with $\omega = \arctan(\|n\|/d)$, confirming two degenerate TT pairs and dimension four.

3.2 Chiral single-cone lift and dispersion

A direct positive lift that squares the signed boundary carrier converts the $\pm$ wall partners into two positive frequencies and yields four TT branches. The decisive step avoids the square. Define the positive scalar stiffness

$$K_{\mathrm{ch}}(k) = D(k)^\dagger D(k) + \|n(k)\|^2 I, \qquad (D\psi)_s = \psi_{s+1} - r(k)\psi_s, \quad (D\psi)_{W-1} = c(k)\psi_{W-1}, \tag{3}$$

with $r(k) = 0.42 + 2.5\, m_W(k)$, $c(k) = 2\, m_W(k)$, and the Wilson profile $m_W(k) = \frac{1}{4}(3 - \cos k_x - \cos k_y - \cos k_z)$.

Proposition 2 (Single cone, gapped doublers, $z = 1$)**.** $D^\dagger D$ *has exactly one soft mode, localized on the lower wall, with a finite second gap. The Wilson profile gaps the lattice doubler even where* $\|n\| \to 0$*. The surviving boundary oscillator inherits the linear dispersion of the connection norm,* $\omega(k) = \|n(k)\| + O(|k|^3)$*.*

Audit. At $W = 48$: a single eigenvalue at machine zero, second gap $\sqrt{\lambda_2} = 0.5815483681$ (ten-digit match to the ledger), soft mode right-wall weight $\sim 10^{-19}$ (exact chirality); at the coined-doubler momentum $\|n\| \approx 10^{-9}$ but the lowest frequency is 1.406 (gapped). The infrared exponent is $z = 0.999787$ with $R^2 = 0.9999999$, and the lowest frequency equals $\|n(k)\|$ to five parts in 10^5. This last equality is the interpretive crux: the $z = 1$ is *carried* by the connection norm; the chiral factorization supplies positivity, single-cone selection, doubler removal, and finite residue, not the exponent itself.

3.3 Conserved-source response and Ward identities

Couple a source S_{ij} to the boundary oscillator. The static susceptibility is $\chi(k) = \sum_m Z_m(k)/\lambda_m(k)$ with Z_m the boundary overlaps and λ_m the stiffness eigenvalues.

Proposition 3 (Correct massless response to conserved sources)**.** *For an externally conserved (transverse) TT source, the carrier exhibits the static graviton susceptibility $\chi(k) \sim |k|^{-2}$, a dynamical pole at $\omega = \|n\|$ with finite residue, and TT (trace and divergence) Ward identities at numerical precision.*

Audit. Independent re-implementation gives static exponent -1.999525, dynamical exponent 0.999762, residue plateau $\lambda_0\chi = 0.8235980$ (constant to 5×10^{-6} across the infrared), trace-projector Ward residual 2.5×10^{-16}, and divergence-projector residual 1.3×10^{-18}. The $1/k^2$ is structurally tied to the massless pole ($\chi \approx Z_0/\lambda_0$ with $\lambda_0 = \|n\|^2$): it certifies that the pole is a genuine massless pole with finite residue, which is the nontrivial content.

4 Three faces of one obstruction

4.1 Face 1 — finite dimension forces $z = 3$

Any exact compact local-*character* realization in fixed local dimension is governed by Observation 1: the minimal local TT characters are high degree, so the physical kinetic and potential blocks scale as $O(k^2)$ and $O(k^4)$, and the symplectic pairing of the surviving logicals is homogeneous of degree three with rank two everywhere off the origin. The dispersion is then locked at $\omega \sim |k|^3$.

Proposition 4 (Lifshitz lock)**.** *For the exact compact local-character spin-2 pairing in three dimensions, the TT dispersion exponent is $z = 3$. No bounded-range polynomial canonical change of variables lowers it; the canonical inverse is singular as $|k|^{-3}$ and is nonlocal.*

Audit. The symplectic form $\Omega(d) = Z(d)X(d)^\top$ is homogeneous degree three; its minor-sum-of-squares factors into a product of positive-definite quadratics times a manifestly positive sextic, so $\operatorname{rank}\Omega = 2$ for all nonzero d (numerically bounded below by 0.25 on the unit sphere). Random positive Hessians give $z = 3.000000$ across 1200 branches, and the Schur complement $B - C^\top A^{-1} C = O(k^4)$, so integrating out auxiliaries does not rescue an $O(k^2)$ stiffness. Independently, no exact finite-dimensional canonical code satisfies $[Q, P] = iI$: the normalized residual is bounded below by one for every code dimension, since $\operatorname{tr}[Q, P] = 0 \neq \operatorname{tr}(iI)$. The chiral carrier evades this face only by abandoning finite local dimension—its boundary oscillator is genuinely continuous-variable.

4.2 Face 2 — the masslessness is not protected

Proposition 5 (First class but relevant)**.** *A positive local first-class mass exists and is relevant. Adding $m^2 I$ to K_{ch} preserves the constraint ideal yet gaps the infrared: $\omega(k) = \sqrt{m^2 + \|n\|^2} \to m$ as $k \to 0$. Masslessness is therefore fine-tuned, not enforced by first-classness.*

Audit. A generic positive copy-space block preserves the constraints to 2×10^{-15} (so the mass is genuinely first-class), while the deep-infrared exponent collapses $z \to 0$ at fixed m^2 with crossover $k_* = \sqrt{m^2}/c_g$ ($m^2 = 10^{-6} \Rightarrow k_* = 10^{-2}$, gap 10^{-3}). The absence of a protecting symmetry is Observation 1 again: there is no low-order local gauge-invariant operator whose conservation would forbid the mass.

4.3 Face 3 — local sources couple at Cotton order

Proposition 6 (Source-locality suppression)**.** *The $1/k^2$ response of Proposition 3 requires an externally conserved source. A strictly local polynomial source that is exactly TT by itself first appears at Cotton order, so its response is derivative-suppressed as $O(k^4)$ rather than $1/k^2$.*

Audit. With the Cotton-order form factor $\|n\|^3$ (Observation 1), the local-source response scales as $\|n\|^6 \chi \sim k^4$; the audited exponent is 3.999. We flag that in the present bundle this enters as an asserted proxy from the local-TT-module statement, not as an independent recomputation; its justification is Proposition 4's character-degree count. Universal coupling to local stress-energy—the defining feature of gravity—is therefore *not* established by the carrier.

5 Synthesis and what a graviton phase requires

Propositions 4–6 are one statement. Observation 1—no low-derivative local gauge-invariant TT operator—forces high-degree characters (Face 1), forces local-source form factors to Cotton order (Face 3), and removes any low-order gauge-invariant quantity whose conservation could forbid a mass (Face 2). The chiral continuous-variable carrier is precisely the construction that defeats Face 1 by leaving finite local dimension and removing the doubler; it cannot, by the same token, defeat Faces 2 and 3, which do not depend on local dimension.

Observation 2 (What must be added)**.** *A protected emergent graviton phase requires an* external *input that the linear carrier cannot supply internally: a matter sector whose diffeomorphism Ward identity makes its stress tensor exactly conserved (resolving Face 3 by providing conserved sources) and whose symmetry forbids the relevant mass (resolving Face 2). Equivalently, the masslessness and the universal coupling must be* consequences of a gauged symmetry*, not properties tuned into a free carrier.*

This reframes the remaining program. The next quantitative target is not to chase an exact fixed-dimensional massless phase—Proposition 4 shows that object does not exist in the compact-character class, and Proposition 5 shows even the continuous-variable version is unprotected. The tractable target is the *approximation law*: at finite local dimension d (or bond dimension χ), how do the residual gap, the TT residue, and the Ward residuals scale? The truncation audits already in hand (worst-site occupation $\bar{n} \approx 0.171$; onsite dimensions $d = 8, 10, 12$ for tails below $10^{-6}, 10^{-8}, 10^{-10}$) supply the diagnostics. A clean statement of the form "bond dimension χ buys graviton fidelity $\epsilon(\chi)$ with residual gap $\Delta(\chi)$" would convert the present negative results into a positive, falsifiable scaling theory, which is the honest and achievable next paper.

Reproducibility

The numerical statements above were regenerated from independent re-implementations of the audited models (constraint algebra, chiral stiffness K_{ch}, conserved-source susceptibility, mass perturbation, and the compact-character symplectic pairing), not solely from the original audit scripts. The first-class algebra, single-cone spectrum, $1/k^2$ response, Ward residuals, mass relevance, and $z = 3$ Lifshitz lock each reproduced to the precision quoted. The Cotton-order suppression (Face 3) is the one statement carried as an asserted proxy rather than recomputed, as noted at Proposition 6.