QATNU finite-dimension and Ward approximation-law audit

Executive verdict

The chiral continuous-variable spin-2 carrier has a strong finite-dimensional local approximation law, but not an exact fixed-d protected many-body phase. Fixed d controls local IR observables exponentially well. Exact global no-leakage, positive onsite protection, and finite-range TT source Ward accuracy each have their own resource growth law.

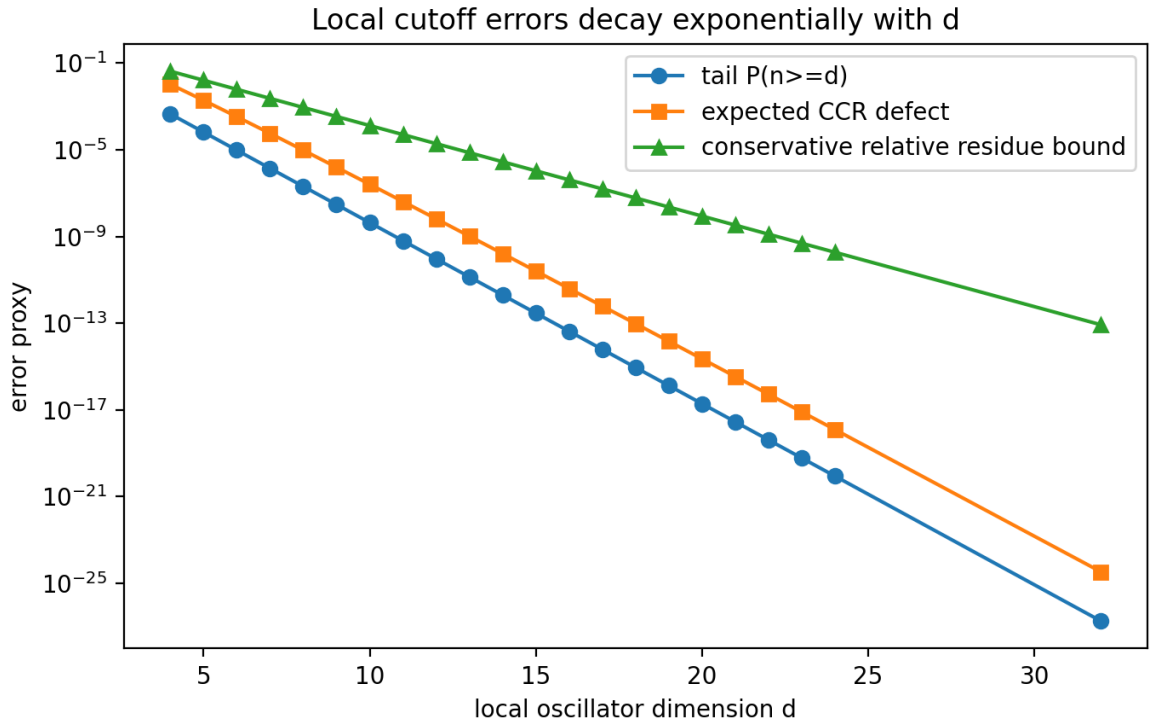
Quantity	Value
eta, worst-site thermal ratio	0.145968795676762
TT residue Z_inf	0.823598006357965
dynamic z from source audit	0.999761765186686
static response exponent	-1.999525292313153
IR velocity prefactor c	0.099609932404754
guard gap law	Delta = 2.372130 sqrt(lambda delta_CCR), R2=0.9999999762

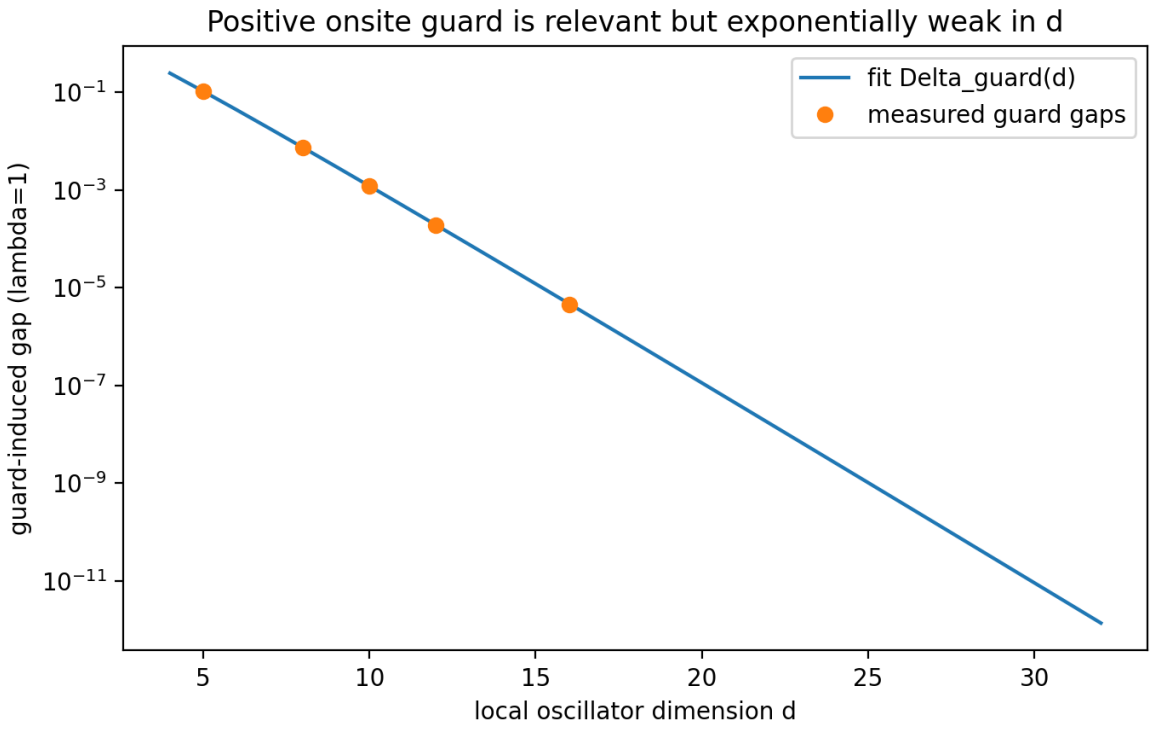
Local finite-d law

The optimized local oscillator basis has a geometric tail. With eta fixed by the worst-site covariance extrapolant:

`P_tail(d)=eta^d; delta_CCR(d)=d(1-eta)eta^(d-1); Delta_guard(d,lambda)=C sqrt(lambda delta_CCR(d)).`

d	P_tail	delta_CCR	Delta_guard	L_star	Z lower bound
5	6.626743e-05	0.00193858	0.104443	5.99242	0.810189
8	2.061010e-07	9.646814e-06	0.00736768	84.9478	0.82285
10	4.391372e-09	2.569295e-07	0.00120239	520.519	0.823489
12	9.356648e-11	6.569242e-09	1.922632e-04	3255.27	0.823582
16	4.247764e-14	3.976437e-12	4.730266e-06	1.323113e+05	0.823598
20	1.928415e-17	2.256546e-15	1.126836e-07	5.554205e+06	0.823598
24	8.754686e-21	1.229322e-18	2.630094e-09	2.379640e+08	0.823598

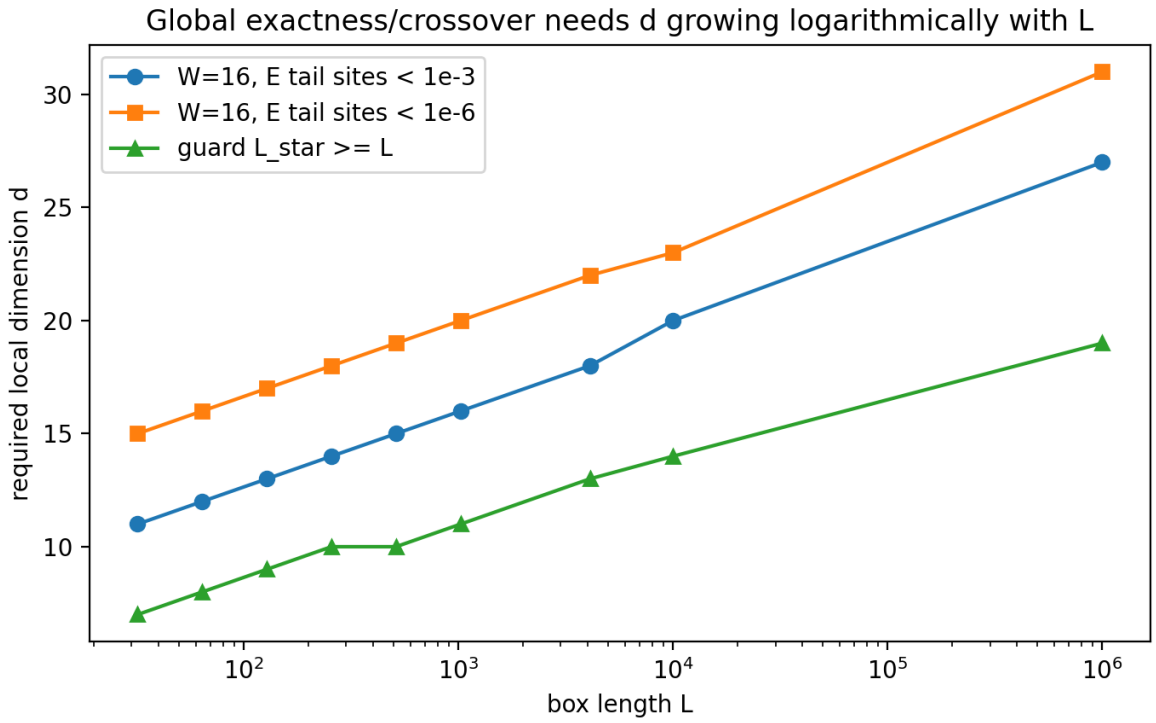

Local cutoff errors decay exponentially with d
tail P(n>=d)
expected CCR defect
conservative relative residue bound
error proxy
10^{-1}
10^{-5}
10^{-9}
10^{-13}
10^{-17}
10^{-21}
10^{-25}
5
10
15
20
25
30
local oscillator dimension d

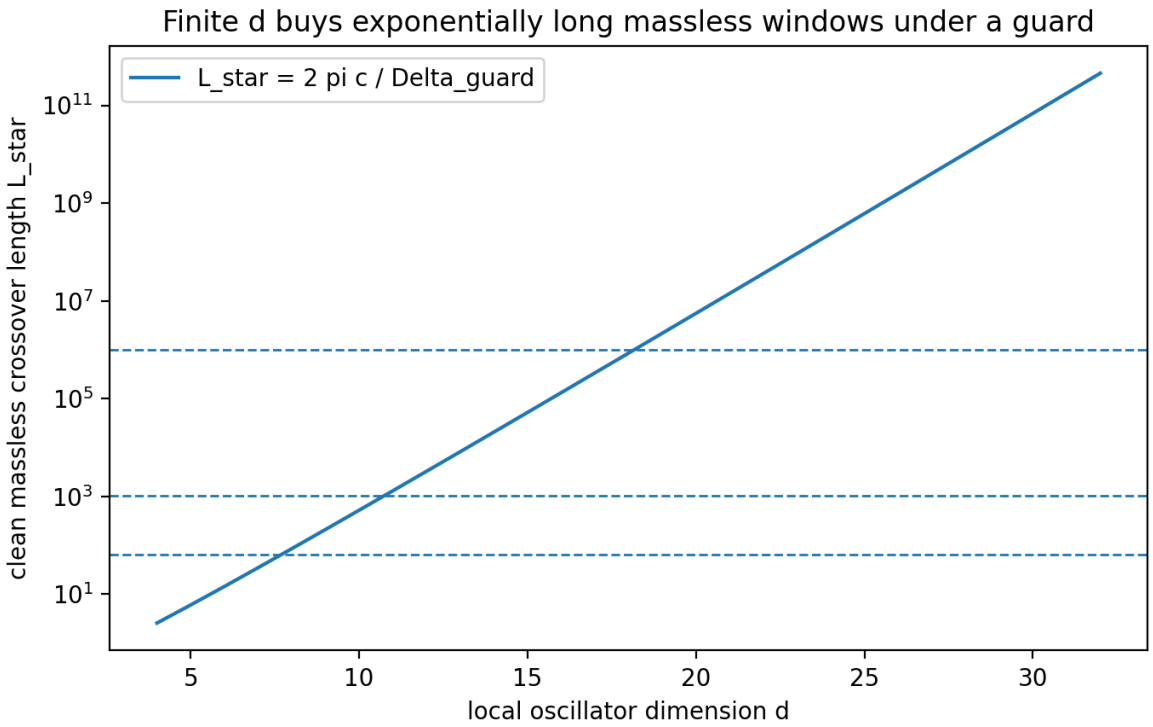

Positive onsite guard is relevant but exponentially weak in d
fit Delta_guard(d)
measured guard gaps
guard-induced gap (lambda=1)
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
5
10
15
20
25
30
local oscillator dimension d

Global no-leakage and finite-size windows

Local finite-d control is not the same as an exact global code. For W local synthetic oscillators per spatial site, E[N_tail]=W L^3 eta^d. Hence d grows logarithmically with L for global no-leakage. A positive onsite guard creates a small mass; the massless cone is visible only below L_star(d)=2 pi c/Delta_guard(d).

L	d: E tail<1e-3	d: E tail<1e-6	d: guard L_star>=L
64	12	16	8
128	13	17	9
256	14	18	10
512	15	19	10
1024	16	20	11
4096	18	22	13
1e+06	27	31	19

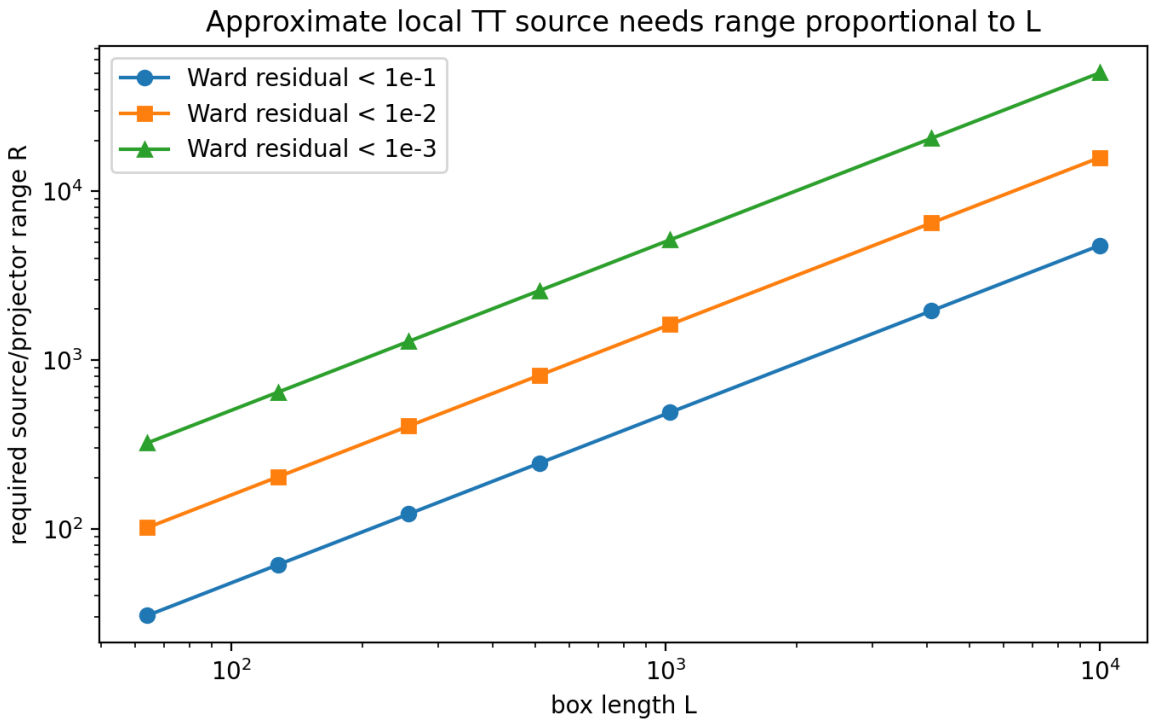



Finite d buys exponentially long massless windows under a guard
L_star = 2 pi c / Delta_guard
clean massless crossover length L_star
10^{11}
10^{9}
10^{7}
10^{5}
10^{3}
10^{1}
5
10
15
20
25
30
local oscillator dimension d

Finite-range Ward/source approximation law

The obstruction paper reframes the source issue as absence of a low-derivative exactly local TT operator. A finite-range approximation to the nonlocal TT source/projector can be modeled by a Yukawa regulator with m=1/R. At the lowest box momentum k_min=2 pi/L, the normalized Ward residual is m^2/(k_min^2+m^2), so fixed Ward accuracy requires R proportional to L.

L	R for Ward<1e-1	R for Ward<1e-2	R for Ward<1e-3	residual if R=L
64	30.5577	101.349	321.946	0.0247045
128	61.1155	202.697	643.892	0.0247045
256	122.231	405.394	1287.78	0.0247045
512	244.462	810.789	2575.57	0.0247045
1024	488.924	1621.58	5151.13	0.0247045
4096	1955.7	6486.31	2.060454e+04	0.0247045
10000	4774.65	1.583572e+04	5.030404e+04	0.0247045



Conclusion

The approximation law is now sharp. The CV chiral carrier is exact and local. Finite local dimension is an excellent controlled regularization for local low-energy observables. But fixed-d positive guards and finite-range source compilers do not by themselves produce a thermodynamic protected graviton phase. The next exact route must introduce a real matter/Ward/topological symmetry that forbids the mass term and supplies conserved TT coupling, rather than relying on another cutoff penalty.