One Obstruction, Four Faces: A Chiral Bosonic Spin-2 Carrier and the Barrier to an Emergent Graviton Phase

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June 2026

Abstract

We construct and audit a local, positive, first-class chiral bosonic carrier for transverse-traceless (TT) spin-2 modes, and we use it to isolate what separates a working linear spin-2 *carrier* from a symmetry-protected emergent *graviton phase*. The carrier is manifestly positive, has exactly two TT branches, a single boundary cone with finite residue, linear dispersion, and—probed by an externally conserved TT source—the static $1/k^2$ susceptibility with TT Ward identities at machine precision. We then show that four apparently distinct obstacles to promoting this object to a graviton are a single algebraic fact seen from four sides: there is no low-derivative-order local gauge-invariant spin-2 operator. That one fact (i) locks the dispersion of any exact compact finite-dimensional local realization to Lifshitz $z = 3$; (ii) Cotton-suppresses the coupling to a strictly local TT source to $O(k^4)$; (iii) leaves masslessness unprotected, since a positive local first-class mass is relevant; and (iv) forces the single chiral cone to violate the topological charge-sum rule of a local periodic bulk. The carrier evades face (i) by leaving finite local dimension and evades face (iv) by spending a synthetic dimension, but faces (ii) and (iii) survive because they do not depend on either resource. A finite-resource approximation law makes the cost of each face precise: local infrared observables converge exponentially in the onsite cutoff, global no-leakage costs grow only logarithmically in system size, and Ward/source exactness is the sole cost that grows linearly. We conclude that the remaining exact route requires a real gauged (matter-Ward or topological) symmetry, not a further cutoff. We are explicit throughout about which statements are theorems, which are numerics, and which are calibrated models. In particular the linear dispersion is *inherited* from the input connection; the construction supplies positivity, locality, single-cone selection, and finite residue, not the exponent.

1 Claim boundary

This paper does not claim emergent gravity. It claims two things and separates them cleanly. First, a *linear spin-2 response carrier*: a local, positive, first-class architecture whose constrained infrared sector contains exactly two TT branches, realized as a single chiral boundary cone with finite residue and the correct conserved-source response. Second, an *obstruction map*: a sharp, mostly negative account of the distance from that carrier to a symmetry-protected graviton. The negative half is the part we most want believed. A companion scalar construction using the identical matter-controlled-bilinear architecture completes cleanly precisely because its target carries none of the obstructions below; the contrast localizes the difficulty in the spin-2 *target*, not in the method.

Throughout, "verified" means regenerated from an independent re-implementation, not only from the original audit code.

2 The unifying fact

Represent a local spin-2 operator on a translation-invariant lattice as a polynomial in the lattice momentum k. The physically meaningful operators are gauge invariant: they survive the linearized diffeomorphism (Stueckelberg) quotient and are transverse-traceless. The organizing statement concerns the minimal polynomial degree of such operators.

Observation 1 (No low-order local gauge-invariant TT operator)**.** *In the linearized 3D tensor module there is no nonzero local TT operator through derivative order two. By exact symbolic enumeration, the physical character module has zero elements at degree* 0*, zero Z-type characters at degree* 1*; the minimal X-type character is degree* 1 *and the minimal Z-type character is degree* 2*. The first strictly local TT creator enters at Cotton order (third derivative order). The transverse projector is degree zero but nonlocal.*

Observation 1 is elementary—the Cotton tensor is the standard 3D conformal curvature—but it is the common root of every obstruction in Section 4. We first record what the carrier achieves.

3 The chiral bosonic carrier

3.1 First-class Stueckelberg quotient

The variables are symmetric tensors A_{ij} (coordinates) and B_{ij} (momenta) with one Stueckelberg pair (ϕ, π): fourteen real dimensions per momentum. For a local connection vector $n(k)$ define

$$T = \operatorname{tr} A + \phi, \qquad D = n_i n_j A_{ij}, \qquad L_j = n_i B_{ij} - n_j \operatorname{tr} B + 2 n_j \pi. \tag{1}$$

Proposition 1 (Abelian first class)**.** *For every nonzero n the five constraints* (1) *are first class: all ten Poisson brackets vanish. The quotient reduces the fourteen-dimensional phase space to four, i.e. exactly two TT canonical pairs, removing the helicity-*0 *and helicity-*$\pm$1 *sectors.*

Audit. The maximum bracket is 2.8×10^{-17} at generic n and 3.6×10^{-15} worst over 2000 random connections; the trace–momentum bracket from $\operatorname{tr} A$ cancels against the Stueckelberg bracket $2 n_j \pi$, and the D–L_j bracket cancels between $n_i B_{ij}$ and $-n_j \operatorname{tr} B$. The lifted map is symplectic to $\sim 10^{-15}$ with reduced quasienergies $(-4\omega, -4\omega, +4\omega, +4\omega)$.

3.2 Chiral single-cone lift

A positive lift that squares the signed carrier converts the wall partners into two positive frequencies and yields four TT branches. The decisive step avoids the square. Define the positive stiffness

$$K_{\mathrm{ch}}(k) = D(k)^{\dagger} D(k) + \|n(k)\|^2 I, \quad (D\psi)_s = \psi_{s+1} - r(k)\psi_s, \ (D\psi)_{W-1} = c(k)\psi_{W-1}, \tag{2}$$

with $r = 0.42 + 2.5\, m_W$, $c = 2\, m_W$, $m_W(k) = \frac{1}{4}(3 - \cos k_x - \cos k_y - \cos k_z)$. Equivalently, at the signed-band level the carrier is a Kaplan domain wall

$$H_{\mathrm{DW}}(k, s) = n_i \Gamma_i + [M_0 + \beta m_W(k) + 1]\Gamma_5 + \big[-\tfrac{1}{2}\Gamma_5 - \tfrac{i}{2}\Gamma_4 \big]_{\langle s, s+1 \rangle} + b_{\mathrm{top}} \Gamma_1 \, \delta_{s, W-1}, \tag{3}$$

with $M_0 = -1$, $\beta = 10$, $b_{\mathrm{top}} = 2$, $W = 24$.

Table 1: Verified carrier diagnostics (independent re-implementations).

quantity	value	role
first-class bracket (max / worst-of-2000)	2.8×10^{-17} / 3.6×10^{-15}	2 TT pairs
K_{ch} second gap (W=48)	0.5815483681	single cone
doubler lowest frequency ($\|n\| \approx 10^{-9}$)	1.406	Wilson removal
H_{DW} branch count (with / without wall mass)	2 / 4	mirror removed
dispersion exponent z	0.999787	inherited (§3)
static susceptibility exponent	-1.999525	$1/k^2$ response
TT Ward residual (trace / divergence)	2.5×10^{-16} / 1.3×10^{-18}	gauge exact
boundary residue Z_∞	0.823598	finite

Proposition 2 (Single cone, gapped doublers, wall-localized)**.** *K_{ch} has exactly one soft mode, and H_{DW} has exactly one boundary cone. The Wilson profile gaps the lattice doubler even where $\|n\| \to 0$. The upper-wall mass removes the mirror branch: the low TT branch count is 2 with the wall mass and 4 without.*

Audit. At $W = 48$, K_{ch} has a single machine-zero eigenvalue with second gap $\sqrt{\lambda_2} = 0.5815483681$ and right-wall weight $\sim 10^{-19}$; at the coin-doubler momentum $\|n\| \approx 10^{-9}$ but the lowest frequency is 1.406 (gapped). An independent build of H_{DW} with a distinct Clifford representation gives 2 soft branches with b_{top} and 4 without, the surviving mode carrying lower-wall weight 1.000000 and upper-wall weight $< 10^{-15}$.

3.3 Dispersion, inherited

Proposition 3 (Linear dispersion from the connection norm)**.** *The surviving cone has $\omega(k) = \|n(k)\| + O(|k|^3)$, with $z = 0.999787$ ($R^2 = 0.9999999$).*

Audit and interpretation. The lowest frequency equals $\|n(k)\|$ to five parts in 10^5 across the infrared. This is the interpretive crux and we state it plainly: the $z = 1$ is *carried* by the connection norm, which is an input from the underlying coined walk; the chiral factorization supplies positivity, single-cone selection, doubler removal, and finite residue, not the exponent. Claims of the form "an emergent $z = 1$ graviton" would misattribute an input to the construction.

3.4 Conserved-source response and Ward identities

Proposition 4 (Correct massless response to conserved sources)**.** *For an externally conserved TT source the carrier exhibits the static susceptibility $\chi(k) \sim |k|^{-2}$, a pole at $\omega = \|n\|$ with finite residue, and TT trace and divergence Ward identities at numerical precision.*

Audit. Independent re-implementation gives static exponent -1.999525, dynamical exponent 0.999762, residue plateau $\lambda_0 \chi = 0.823598$ (constant to 5×10^{-6}), trace Ward residual 2.5×10^{-16}, divergence Ward residual 1.3×10^{-18}. Since $\chi \approx Z_0/\lambda_0$ with $\lambda_0 = \|n\|^2$, the $1/k^2$ certifies a genuine massless pole with finite residue; it is a consistency check on the pole, not independent new physics.

4 Four faces of one obstruction

4.1 Face 1: finite dimension forces $z = 3$

Proposition 5 (Lifshitz lock)**.** *For the exact compact local-character spin-2 pairing in three dimensions the TT dispersion exponent is $z = 3$. No bounded-range polynomial canonical change of*

variables lowers it, and no exact finite-dimensional local canonical code exists.

Audit. The symplectic pairing $\Omega(d) = Z(d)X(d)^\top$ is homogeneous of degree three; its minor-sum-of-squares factors into positive-definite quadratics times a manifestly positive sextic, so $\operatorname{rank}\Omega = 2$ for all nonzero d (bounded below by 0.25 on the unit sphere). Random positive Hessians give $z = 3.000000$ over 1200 branches, and the Schur complement $B - C^\top A^{-1} C = O(k^4)$: integrating out auxiliaries does not rescue an $O(k^2)$ stiffness. Independently, $\operatorname{tr}[Q, P] = 0 \neq \operatorname{tr}(iI)$ forbids $[Q, P] = iI$ in any finite dimension, with normalized residual bounded below by one. By Observation 1 the physical kinetic and potential characters start at $O(k^2)$ and $O(k^4)$, which is the origin of the lock. The carrier evades this face only by abandoning finite local dimension: its boundary oscillator is genuinely continuous-variable.

4.2 Face 2: masslessness is not protected

Proposition 6 (First class but relevant)**.** *A positive local first-class mass exists and is relevant. Adding $m^2 I$ to K_{ch} preserves the constraint ideal yet gaps the infrared, $\omega = \sqrt{m^2 + \|n\|^2} \to m$; masslessness is fine-tuned, not enforced.*

Audit. A generic positive copy-space block preserves the constraints to 2×10^{-15} (the mass is genuinely first class), while the deep-infrared exponent collapses $z \to 0$ at fixed m^2 with crossover $k_* = m/c_g$. The absence of a protecting symmetry is Observation 1 again: no low-order local gauge-invariant quantity exists whose conservation would forbid the mass.

4.3 Face 3: local sources couple at Cotton order

Proposition 7 (Source-locality suppression)**.** *The $1/k^2$ response of Proposition 4 requires an externally conserved source. A strictly local polynomial source that is exactly TT by itself first appears at Cotton order, so its response is derivative-suppressed as $O(k^4)$.*

Audit and scope. With the Cotton-order form factor $\|n\|^3$ (Observation 1), the local-source response scales as $\|n\|^6 \chi \sim k^4$; the audited exponent is 3.999. We flag that this enters as an asserted proxy from the local-TT-module statement, not as an independent recomputation; its justification is the character-degree count of Face 1. Universal coupling to local stress-energy—the defining feature of gravity—is therefore not established by the carrier.

4.4 Face 4: a lone chiral cone violates charge conservation

The connection map $n : T^3 \to \mathbb{R}^3$ has isolated zeros (nodes), each carrying a topological charge equal to the degree of $\hat{n}$ on a small enclosing sphere.

Proposition 8 (Doubling is a conservation law)**.** *The coined walk has exactly two nodes, at the origin and a diagonal point, carrying equal and opposite charge; both are locally isotropic mirror cones. Within locality, translation invariance, a finite-range bulk symbol, Hermiticity, and preservation of the origin cone, a single bulk cone cannot be obtained by smooth local deformation.*

Audit. Two independent methods—$\operatorname{sign}(\det J)$ and the solid-angle degree of $\hat{n}$—give opposite unit charges summing to zero (the absolute sign is an orientation convention). The Jacobians are $\sin\alpha$ times orthogonal matrices with opposite determinant sign; $\det J = \pm 9.950108 \times 10^{-4}$, singular values 0.0998334. This is Nielsen–Ninomiya, i.e. Poincaré–Hopf for a map from the boundaryless 3-torus. A topological charge cannot be deleted by continuous deformation of the coin; the domain wall of (3) does not delete the partner but *relocates* the opposite charge to the other synthetic boundary

and gaps it from the selected wall ($m_W = 0$ at the origin cone, $m_W = 0.833$ at the doubler). We state the assumptions explicitly: the statement is evaded by nonlocal projection, interactions or topological order, cone-destroying pair annihilation, or—as the carrier itself does—by spending an explicit synthetic dimension. The domain wall is not the unique resolution; it is the resolution that keeps locality and Hermiticity while paying with a dimension.

5 The approximation law

Faces 1–4 rule out an *exact* fixed-dimensional protected graviton. What remains is a controlled approximation, and its cost is governed by a single measured constant: the soft-mode mean occupation $\bar{n} = 0.171$, i.e. $\eta = \bar{n}/(1+\bar{n}) = 0.145969$. Three costs separate sharply.

Proposition 9 (Cost trichotomy)**.** *Local infrared observables converge exponentially in the onsite cutoff d; global no-leakage costs grow logarithmically in system size L; Ward/source exactness is the sole cost that grows linearly in L.*

Audit. **Local (exponential).** The truncation tail is $P_{\text{tail}}(d) = \eta^d$ and the canonical-commutator defect is $\delta_{\text{CCR}}(d) = d(1-\eta)\eta^{d-1}$; $d = 10$ gives tail 4×10^{-9}. A positive onsite guard produces a mass $\Delta = C\sqrt{\lambda\,\delta_{\text{CCR}}}$ (fitted $C = 2.372$; the $\sqrt{\ }$ is generic perturbation response, the constant is calibrated), giving a finite-size massless window $L_* = 2\pi c/\Delta$—not a thermodynamic phase. **Global (logarithmic).** With $E[N_{\text{tail}}] = WL^3\eta^d$, global fidelity needs $d \sim 3\log L/|\log\eta|$; $d = 27$ suffices for $L = 10^6$. **Ward (linear).** A finite-range approximation to the nonlocal TT source compiler, modeled as a Yukawa regulator $m = 1/R$, has normalized residual $m^2/(k_{\min}^2 + m^2)$ at $k_{\min} = 2\pi/L$; fixed residual requires $R \propto L$, and at $R = L$ the residual is the L-independent constant $1/(4\pi^2+1) = 0.02470$. Every table row of the underlying audit reproduces to the quoted precision. The exponents (exponential, logarithmic, linear) are robust; the two prefactors (C and the Yukawa form) are calibrated models of the guard and compiler, respectively.

The reading is unambiguous. Local physics is cheap, global bookkeeping is cheap, and the *only* non-scalable cost is Ward/source exactness—which is Faces 2 and 3 reappearing as a scaling law. It cannot be bought down with a further cutoff.

6 Synthesis: four debts, four currencies

Observation 1 is one refusal of locality, and each face is the bill in a different currency:

- **Character debt** (Face 1): no low-order local gauge-invariant TT character $\Rightarrow z = 3$ in finite-dimensional compact codes. Paid by continuous variables (and, at the level of stiffness, by dressing a non-character term).
- **Canonical debt** (Face 1): $[Q, P] = iI$ has no finite-dimensional representation. Paid by continuous variables; at fixed d it reappears as $\delta_{\text{CCR}}(d)$.
- **Symmetry debt** (Faces 2, 3): no low-order gauge-invariant operator to protect the mass or to make a local source conserved. *Unpaid.* This is the linear cost of Section 5, and the one a cutoff cannot settle.
- **Topological debt** (Face 4): a lone chiral cone violates the charge-sum rule. Paid by a synthetic dimension (the domain wall), which relocates rather than deletes the partner.

The carrier settles the character, canonical, and topological debts and leaves the symmetry debt open. That is exactly why it is a carrier and not a phase. A protected emergent graviton requires the one ingredient the carrier cannot supply internally and the approximation law shows cannot be faked: a real gauged symmetry—a matter sector whose diffeomorphism Ward identity makes its stress tensor conserved (settling Face 3) and whose gauge invariance forbids the relevant mass (settling Face 2). Masslessness and universal coupling must be *consequences of a gauged symmetry*, not properties tuned into a free carrier or propped up by a guard.

This reframes the program's next step. The target is not an exact fixed-dimensional massless phase—Faces 1 and 2 show that object does not exist. The target is a construction in which the diffeomorphism (or a topological-order) symmetry is realized on genuine matter, so that the symmetry debt is discharged by a conservation law rather than deferred to a regulator. The carrier and its obstruction map define precisely what that construction must achieve, and the finite-resource laws quantify what it costs to approximate it until it exists.

Reproducibility

The propositions above were regenerated from independent re-implementations: the constraint algebra, the chiral stiffness K_{ch} and the domain-wall H_{DW} (with an independent Clifford representation), the conserved-source susceptibility and Ward residuals, the mass perturbation, the symplectic degree-three pairing and the $z = 3$ lock, the character-degree enumeration, the node charges by two methods, and every approximation-law table row. Two statements are carried as calibrated models rather than first-principles recomputations, and are flagged in place: the Cotton-order form factor of Face 3 (Proposition 7) and the guard/Yukawa prefactors of Section 5. The dispersion exponent of Proposition 3 is inherited from the input connection and is not a product of the construction.