CORRECTION AND SCOPE

Clock Connections and a Hypothetical Spin-2 Completion

Original title · Condensed Timing Geometry: Clock Connections, Holonomy, and the Spin-2 Completion

The clock connection is additional model data. The proposed spin-2 completion requires additional structure beyond scalar clock measurements.

Manuscript · p. 1, draft 0.3

A Test Protocol for Scalar Timing Closure and Dimensional Selection

Evidence · Scalar source/window residual audit / One-Hamiltonian composition requirements

Condensed Timing Geometry

Clock Connections, Holonomy, and the Spin-2 Completion

Draft 0.3

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Abstract

This paper formulates *Condensed Timing Geometry*, a conservative framework in which observable spacetime is approached through clock relations rather than assumed as a primitive manifold. The central observation is that clock-readable geometry has layers. A scalar clock-load Λ can model local clock-rate changes and redshift-like potential structure, but the relative offsets derived from such a scalar are exact: $A = \mathrm{d}\Lambda$. Exact offsets have zero loop holonomy. A scalar timing field therefore cannot by itself carry curvature, topology, radiation, or a genuine obstruction to global synchronization. The first nontrivial geometric object is a clock-connection: an edge-valued timing 1-cochain A whose loop holonomy obstructs a single global clock assignment.

The framework distinguishes three layers. First, scalar timing is the exact or gravitoelectric-readable sector. Second, timing connections carry local curvature $F = \mathrm{d}A$ and, in the weak-field gravitational analogy, can be identified with the gravitomagnetic g_{0i}-like slice rather than with a new field by default. Third, gravity proper requires a symmetric tensor completion: the transverse-traceless spin-2 part of h_{ij}. We therefore separate local curvature holonomy on contractible loops from flat topological holonomy on non-contractible cycles, and we distinguish Maxwell-like connection radiation from genuine helicity-2 gravitational radiation.

Computational diagnostics are reported only as sanity checks. A discrete connection model shows finite-speed coexact curvature propagation but remains spin-1-like. A scalar-vector-tensor/helicity implementation shows that a symmetric tensor h_{ij} decomposes as $2 + 2 + 2 = 6$, with scalar and vector pieces leaking zero into the TT sector at numerical precision, while the TT helicity phase winds with slope -2 and the vector sector winds with slope -1. These diagnostics do not prove emergent gravity from a microscopic substrate. They identify the necessary mathematical ladder: $\Lambda \to A_i \to h_{\mu\nu} \to h_{ij}^{\mathrm{TT}}$. The claim of this paper is not that gravity has been derived, but that the right observable primitive has been located and the next pass/fail test has become sharp.

Central thesis

Spacetime is not first a manifold. Observable spacetime begins where local clock relations either glue into a global timing order or fail by a measurable holonomy class. Scalar timing is the exact sector; clock-connection holonomy is the first curvature observable; spin-2 gravity requires the tensor completion.

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1 Scope of the framework

Condensed Timing Geometry begins with a simple operational fact: clocks are among the most sensitive probes of geometry. A local scalar timing law can express this sensitivity in the form

$$\omega_{\text{eff}} = \frac{\omega}{1+\alpha\Lambda}, \qquad \frac{\delta\omega}{\omega} \approx -\alpha\Lambda. \tag{1}$$

Here Λ is a scalar clock-load and α is a susceptibility. This layer can describe redshift-like clock slowdown. It is useful, but it is not the whole geometry.

The reason is structural rather than philosophical. A scalar field assigns one value to each vertex or location. It can generate gradients, but those gradients always sum to zero around closed loops. Therefore a scalar timing law cannot encode the failure of global clock synchronization. Curvature-like timing information begins only when relative clock offsets are allowed to live on links or paths as a connection.

The framework therefore separates four roles that are easy to conflate:

Layer	Mathematical object	Role in the framework
Scalar timing	$\Lambda \in C^0$	Local clock-rate shift; redshift-like potential structure.
Timing connection	$A \in C^1$	Relative clock offset; obstruction to global synchronization.
Curvature/holonomy	$F = \mathrm{d}A$, $\oint A$	Local curvature on contractible loops or global/topological timing twist on non-contractible cycles.
Spin-2 completion	h_{ij}^{TT}	True linearized gravitational radiation: two helicity-2 tensor polarizations.

Table 1: The hierarchy used in this paper. The framework does not identify scalar timing with gravity. It treats scalar timing as the first clock-readable layer and then asks what additional structure is required for curvature and gravitational radiation.

This hierarchy is the paper's main discipline. Scalar timing is retained as the exact/potential sector. Pairwise numerical agreement of local patches is not treated as the main invariant, because pairwise agreement can miss global obstruction. The invariant is loop holonomy or, in sheaf language, failure of a global section. Connection dynamics are useful as a toy model for finite-speed curvature propagation, but a 1-form connection remains vector-like. A gravitational claim requires the tensor completion and the transverse-traceless spin-2 test.

The resulting object is not a scalar-only theory of geometry. It is *Condensed Timing Geometry*: a framework in which clock data are organized by cohomological degree and by representation under rotations.

2 From scalar clock load to timing connection

Let K be a finite oriented graph or the 1-skeleton of a finite 2-complex. A scalar clock-load is a 0-cochain

$$\Lambda \in C^0(K;\mathbb{R}). \tag{2}$$

A clock-connection is a 1-cochain

$$A \in C^1(K;\mathbb{R}), \tag{3}$$

assigning an antisymmetric timing offset to each oriented edge:

$$A_{ji} = -A_{ij}. \tag{4}$$

If A is generated by a scalar, then

$$A = \mathrm{d}_0\Lambda, \qquad A_{ij} = \Lambda_j - \Lambda_i. \tag{5}$$

If the complex has faces, the next coboundary maps the connection to curvature:

$$F = \mathrm{d}_1 A. \tag{6}$$

On a face or plaquette p, this is the oriented loop sum

$$F_p = \sum_{e\in\partial p} \sigma(e,p) A_e. \tag{7}$$

Theorem 1 (Scalar timing is holonomy-free)**.** *If a timing offset is generated by a single-valued scalar load, $A = \mathrm{d}_0\Lambda$, then the holonomy around every closed cycle γ vanishes:*

$$\oint_\gamma A = 0. \tag{8}$$

Equivalently, $\mathrm{d}_1 A = 0$ *on every contractible plaquette and* $[A] = 0$ *in first cohomology.*

Proof. For a cycle $\gamma = (i_0, i_1, \ldots, i_n = i_0)$,

$$\oint_\gamma A = \sum_{m=0}^{n-1} (\Lambda_{i_{m+1}} - \Lambda_{i_m}) = \Lambda_{i_n} - \Lambda_{i_0} = 0. \tag{9}$$

The local face version is the cochain identity $\mathrm{d}_1\mathrm{d}_0 = 0$. Thus scalar timing offsets are exact and carry no holonomy. □

This theorem establishes the proper role of the scalar sector: it is not discarded, but it is flat in the precise cohomological sense.

3 Gluing, contextuality, and global synchronization

The right sheaf-like question is not whether adjacent patches have small pairwise numerical mismatch. The right question is whether locally consistent timing charts admit a global clock assignment.

Suppose a probe context P is covered by smaller contexts $\{P_a \to P\}$. Each local chart contains timing data s_a. Pairwise consistency asks that the restrictions agree on overlaps:

$$s_a|_{P_a \cap P_b} = s_b|_{P_a \cap P_b}. \tag{10}$$

But sheaf-theoretic obstruction theory is interesting precisely because pairwise agreement can coexist with global inconsistency. In the Abramsky-Brandenburger framework, contextuality and nonlocality are formulated as obstructions to the existence of global sections over a measurement cover [1]. Cohomological refinements produce an obstruction class whose nonvanishing is a sufficient witness of contextuality [2].

The timing analogue is global synchronization. Local clocks can be synchronized chart by chart, but if the loop holonomy is nonzero, no single global clock potential exists.

Definition 1 (Clock-synchronization obstruction)**.** *For a timing connection $A \in C^1(K;\mathbb{R})$, the synchronization obstruction on a cycle γ is*

$$\mathcal{H}_\gamma(A) = \oint_\gamma A. \tag{11}$$

The obstruction vanishes on all cycles exactly when the connection is globally synchronizable, modulo harmonic/topological qualifications imposed by boundary conditions and the first cohomology of the complex.

Thus the gluing thesis becomes:

$$\boxed{\text{observable timing geometry is global synchronization plus its obstructions.}} \tag{12}$$

4 Local curvature versus topological holonomy

A key precision is that "loop holonomy" has two physically different meanings.

Local curvature holonomy. For a contractible loop bounding a plaquette or small disk, nonzero holonomy measures enclosed curvature:

$$\oint_{\partial p} A = (\mathrm{d}A)_p = F_p. \tag{13}$$

This is the coexact/local curvature sector. It can be localized, radiate, and propagate in a dynamical connection model.

Topological holonomy. For a non-contractible loop, the connection can have nonzero holonomy while $F = 0$ everywhere locally. This is the harmonic sector. It is flat but globally twisted, analogous in form to Aharonov-Bohm-type or Sagnac-type situations.

Both obstruct global synchronization. They should not be conflated.

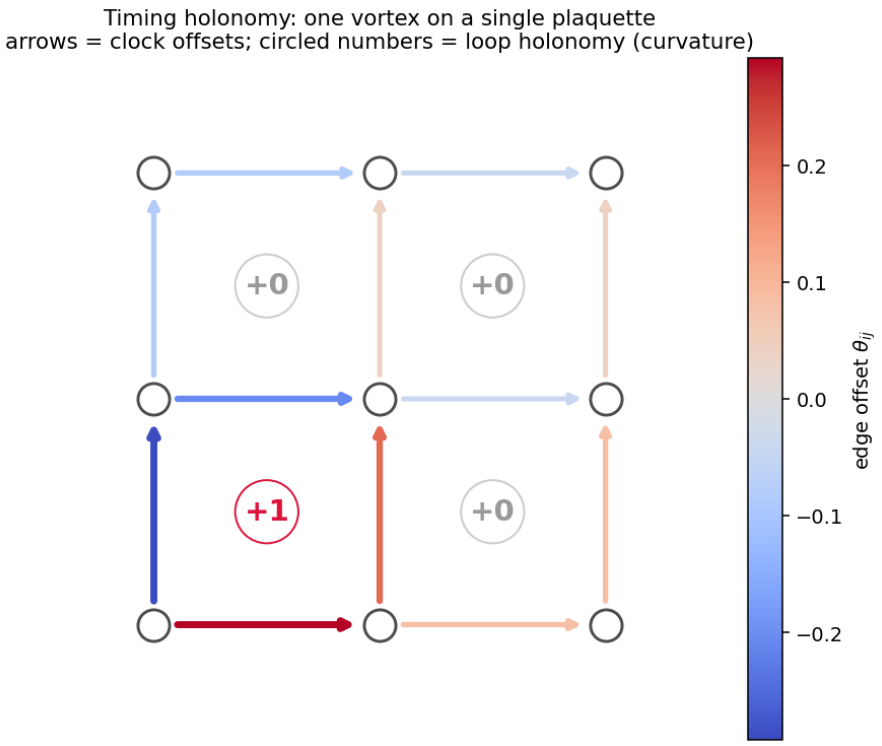

Figure 1: Local curvature holonomy. Edge arrows represent timing offsets A_{ij}. The lower-left plaquette has nonzero $F = \mathrm{d}A$, while neighboring plaquettes are flat. This is a contractible-loop curvature witness, not a non-contractible topological holonomy witness.

This distinction is important for the paper's claim hierarchy. Local curvature can propagate under a wave equation. Harmonic/topological holonomy is typically static in the idealized source-free model.

5 Discrete exterior calculus and the Hodge ladder

Discrete exterior calculus provides the natural finite-complex algebra [3, 4]. The cochain sequence is

$$C^0(K;\mathbb{R}) \xrightarrow{\mathrm{d}_0} C^1(K;\mathbb{R}) \xrightarrow{\mathrm{d}_1} C^2(K;\mathbb{R}). \tag{14}$$

With inner products and adjoints, the 1-form Hodge Laplacian is, up to convention,

$$\Delta_1 = \mathrm{d}_0\mathrm{d}_0^* + \mathrm{d}_1^*\mathrm{d}_1. \tag{15}$$

The Hodge decomposition is

$$C^1(K;\mathbb{R}) = \operatorname{im}\mathrm{d}_0 \oplus \mathcal{H}^1 \oplus \operatorname{im}\mathrm{d}_1^*, \tag{16}$$

where

- $\operatorname{im}\mathrm{d}_0$ is the exact sector: scalar clock gradients;
- $\mathcal{H}^1 = \ker\Delta_1$ is the harmonic sector: flat topological holonomy;
- $\operatorname{im}\mathrm{d}_1^*$ is the coexact sector: local curvature-carrying connection modes.

This gives the first ladder:

$$\Lambda \longrightarrow A_i \longrightarrow F = \mathrm{d}A. \tag{17}$$

The scalar load is the exact/potential shadow. The connection is the first object capable of nontrivial synchronization obstruction.

6 Connection dynamics: useful, but not gravity

A simple wave equation on clock-connections is

$$\frac{\partial^2 A}{\partial t^2} + \Delta_1 A = J. \tag{18}$$

In the source-free case, eigenmodes of Δ_1 with eigenvalues λ_n oscillate with

$$\omega_n = \sqrt{\lambda_n}. \tag{19}$$

This is structurally close to lattice electromagnetism and finite-difference Maxwell/Yee propagation [5]. It is a useful timing-connection toy model, but it is not spin-2 gravity.

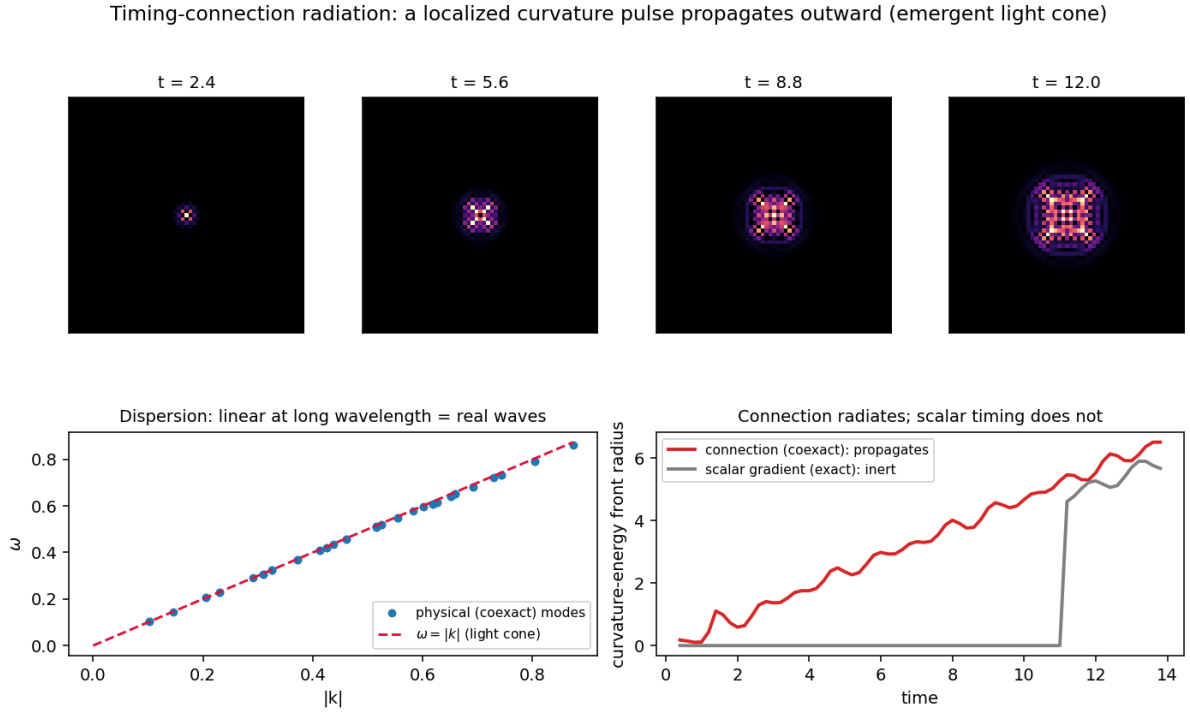

Figure 2: Connection dynamics. A localized coexact timing-curvature pulse propagates outward with finite speed and approximately linear long-wavelength dispersion. The control remains important: scalar-gradient timing kicks are exact and do not radiate curvature except at numerical roundoff. This figure supports the connection/scalar distinction, not an emergent-gravity claim.

The correct conclusion is narrow:

clock-connections can radiate curvature-like waves, but a 1-form wave is spin-1-like. (20)

Gravity requires a further tensor lift.

7 The gravitomagnetic anchor

The clock-connection should not be treated as an unspecified new Maxwell-like field unless forced. Weak-field general relativity already contains a physically meaningful clock-connection-like object: the gravitomagnetic potential, associated with the g_{0i} components of the metric in a gravitoelectromagnetic decomposition. Gravitoelectromagnetism is an approximation to Einstein's equations in weak-field, slow-motion regimes, where mass-energy currents produce gravitomagnetic effects analogous to magnetic fields generated by moving charge [6].

In this language, the scalar timing potential is the gravitoelectric-readable part, while the clock-connection is naturally identified with the gravitomagnetic/time-space part:

$$\Lambda \sim h_{00}, \qquad A_i \sim h_{0i}. \tag{21}$$

This identification is not a derivation from the present toy model. It is a physical anchor that prevents overinterpreting A_i as a new force by default.

Rotating gravitational sources produce frame-dragging/Lense-Thirring effects, experimentally tested by Gravity Probe B and satellite laser-ranging analyses [7, 8]. Related Sagnac and gravitomagnetic clock effects express the same basic fact in clock language: global synchronization around rotating/gravitomagnetic configurations can fail or acquire a loop-dependent timing offset [6, 9, 10].

This is the best physical interpretation of the timing connection:

$$\boxed{A_i \text{ is best read as the gravitomagnetic } h_{0i} \text{ slice, not as gravity itself.}} \tag{22}$$

8 The spin-2 completion

The spin-2 test changes the question. A 1-form connection can have holonomy and finite-speed propagation, but gravitational radiation in linearized GR lives in the transverse-traceless part of a symmetric spatial tensor h_{ij}. For a plane wave propagating along direction $\hat{k}$, the TT conditions are

$$\hat{k}^i h_{ij}^{\mathrm{TT}} = 0, \qquad \delta^{ij} h_{ij}^{\mathrm{TT}} = 0. \tag{23}$$

The two physical polarizations are conventionally called plus and cross. A ring of test particles deforms in a quadrupole pattern, returning to itself under a 180° rotation. This is not how a vector field transforms. The helicity of the radiative tensor mode is ± 2, not ± 1 [11, 12].

Thus the full ladder is

$$\boxed{\Lambda \longrightarrow A_i \longrightarrow h_{\mu\nu} \longrightarrow h_{ij}^{\mathrm{TT}}.} \tag{24}$$

The clock-connection is the bridge from scalar clock lag to metric completion. It is not the endpoint.

8.1 SVT/helicity diagnostic

A scalar-vector-tensor/helicity implementation used an orthonormal basis for the six-dimensional space of symmetric spatial tensors at fixed nonzero wavevector. The decomposition is

$$6 = 2_{\mathrm{scalar}} + 2_{\mathrm{vector}} + 2_{\mathrm{TT}}. \tag{25}$$

The orthonormal construction avoids sector double-counting and makes the scalar, vector, and tensor subspaces mutually clean.

Diagnostic	Result
Scalar sector dimension	2
Vector sector dimension	2
TT sector dimension	2
Total symmetric tensor dimension	6
Maximum Gram off-diagonal error	2.22×10^{-16}
Random tensor reconstruction error	1.33×10^{-16}
Scalar-to-TT leakage	0.0
Vector-to-TT leakage	0.0
Pure-gauge-to-TT leakage	0.0
TT retention error	1.76×10^{-16}
TT helicity phase slope	-2.000
Vector helicity phase slope	-1.000

Table 2: SVT/helicity diagnostic. The result is a numerical sanity check on the decomposition, not a derivation of spin-2 gravity from scalar or connection sectors.

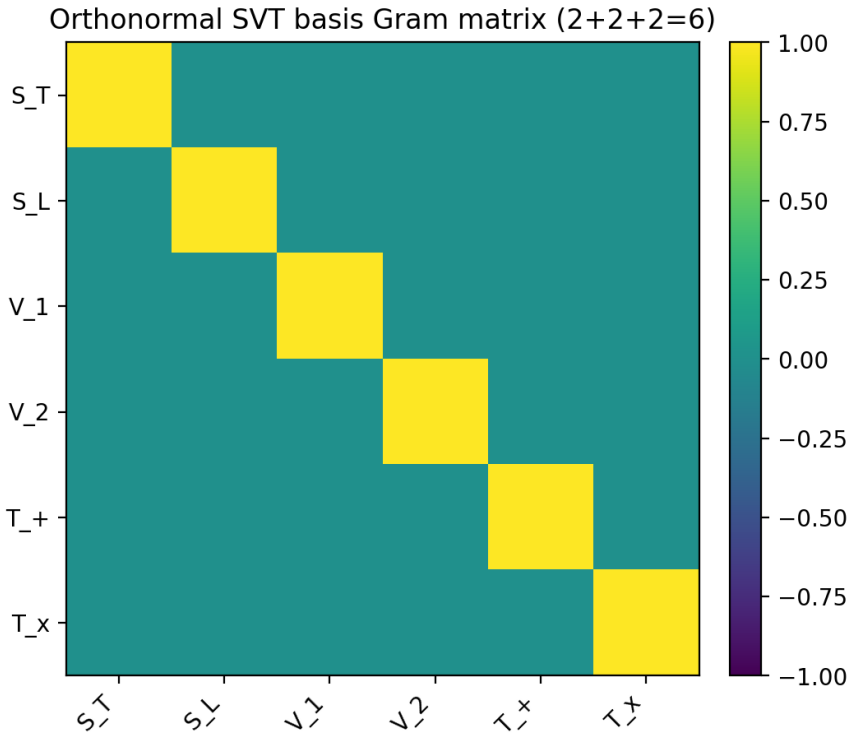

Figure 3: Orthonormal scalar-vector-tensor basis check for a fixed wavevector. The clean $2+2+2 = 6$ split separates scalar, vector, and transverse-traceless tensor degrees of freedom.

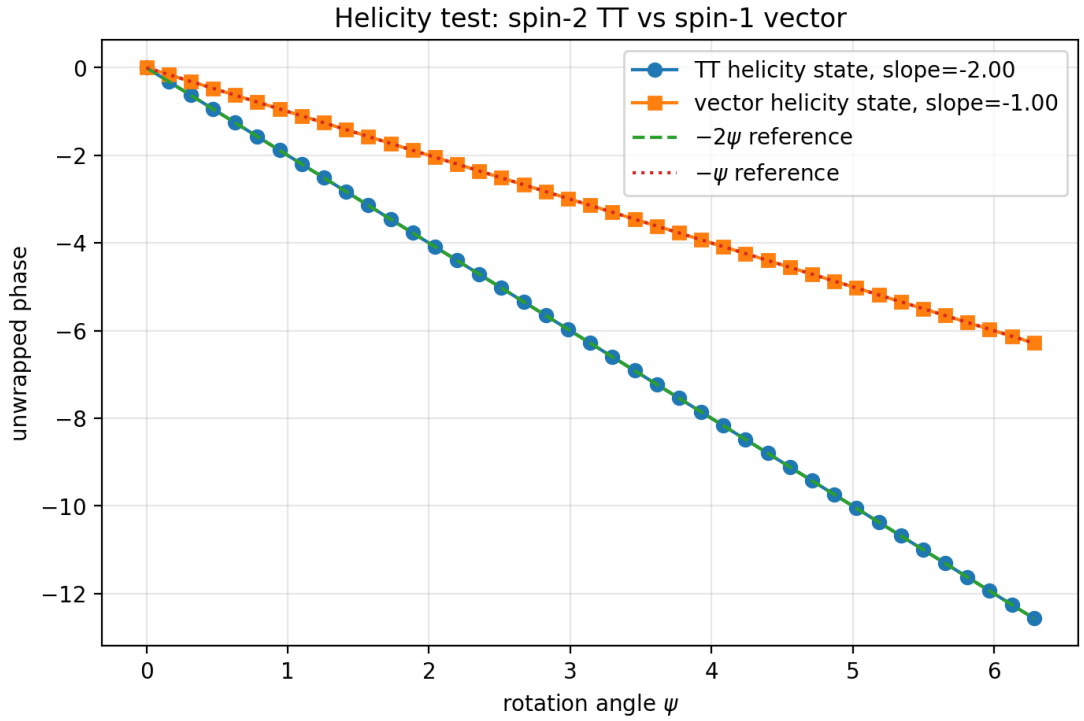

Figure 4: Helicity test. The TT helicity state accumulates phase with slope -2 under rotation about the propagation axis, while a vector helicity state accumulates phase with slope -1. This is the decisive representation-theoretic distinction between the tensor/radiative gravity candidate and the connection/vector sector.

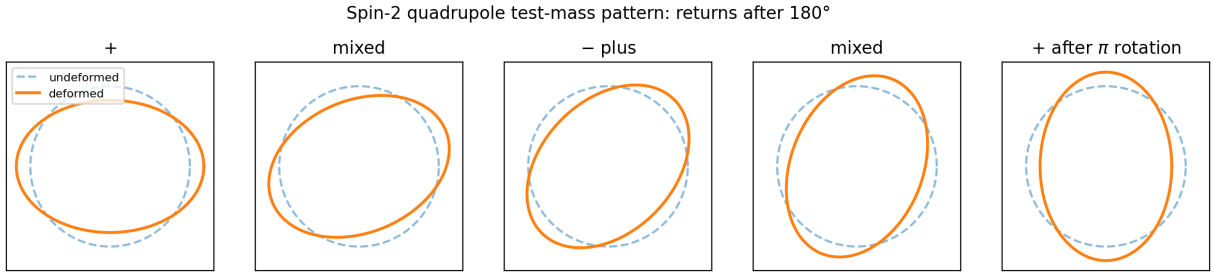

Figure 5: Quadrupole test-mass pattern for a spin-2 TT perturbation. The deformation returns under a 180° rotation, consistent with helicity-2 behavior. This is the visualization that should be used for the spin-2 completion, not for the 1-form timing-connection sector.

The interpretation is strict. This test does not show that a microscopic relational substrate produces h_{ij}^{TT}. It shows what the pass condition looks like once such a tensor field is present. The scalar and vector sectors do not fake it.

9 Claim hierarchy

The framework's strength depends on keeping claims at the right level.

Claim	Meaning	Status
Scalar exactness	$A = \mathrm{d}\Lambda$ gives zero loop holonomy	Theorem-level
Local curvature holonomy	Contractible-loop holonomy equals $F = \mathrm{d}A$	Standard cochain fact
Topological holonomy	Non-contractible loop can have $F = 0$ but nonzero holonomy	Standard Hodge/cohomology fact
Connection radiation	Coexact 1-form dynamics support finite-speed waves	Toy/standard; spin-1-like
GEM anchor	A_i resembles the gravitomagnetic h_{0i} slice	Physical analogy/weak-field anchor
TT spin-2 sector	h_{ij}^{TT} has two helicity-2 modes	Linearized GR fact; numerically checked
Emergence from substrate	A relational/QATNU substrate generates the above hierarchy	Open
New external field	Internal timing data couple to external sensors beyond known channels	Open/experimental
Dimension selection	The observed three-dimensional branch is dynamically selected	Open

Table 3: Claim hierarchy. The paper's established content is mathematical and diagnostic. The emergence and externalization claims remain open.

10 Experimental and numerical program

The revised program has a sharper sequence of tests.

10.1 Exactness control

Generate clock offsets from a scalar load $A = \mathrm{d}\Lambda$. Every loop holonomy should vanish. Any nonzero holonomy indicates hidden connection structure, numerical error, or physics beyond the scalar model.

10.2 Minimal holonomy witness

Use the smallest loop or plaquette to demonstrate local synchronization with global failure. Pairwise overlap residuals should not be treated as sufficient.

10.3 Hodge sector extraction

On a finite complex, decompose measured or simulated timing offsets:

$$A = A_{\mathrm{exact}} + A_{\mathrm{harm}} + A_{\mathrm{coexact}}. \tag{26}$$

Only the coexact component should carry local curvature radiation under the connection wave operator. The harmonic component encodes flat topological holonomy.

10.4 GEM identification

If the connection is physical, test whether its source and coupling match a gravitomagnetic interpretation. A true gravitomagnetic connection is sourced by mass-energy current/angular momentum, not electric charge. If it behaves as an ordinary electromagnetic phase, the result is not new gravity.

10.5 TT spin-2 test

A gravitational claim requires a symmetric tensor completion and a TT projection. The pass conditions are:

1. two TT polarizations;
2. scalar, vector, and pure-gauge leakage into TT is zero within tolerance;
3. helicity phase winds as $e^{\pm 2i\psi}$;
4. long-wavelength dispersion is linear in the relativistic limit;
5. the test-mass pattern is quadrupolar.

11 Conclusion

The strongest result is a hierarchy. Scalar clock load is not the geometry carrier. It is exact and holonomy-free. The first nontrivial observable geometry is a clock-connection, because a connection can fail global synchronization by loop holonomy.

But the connection is not yet gravity. A 1-form connection naturally gives a Maxwell-like or gravitomagnetic-like sector. In the gravitational reading, it is best interpreted as the h_{0i} slice of weak-field geometry. The word *gravity* is earned only at the next layer, where a symmetric tensor field h_{ij} supports a transverse-traceless helicity-2 sector.

The timing hierarchy is therefore:

$$\boxed{\Lambda \text{ (scalar potential)} \to A_i \text{ (clock/GEM connection)} \to h_{\mu\nu} \text{ (metric completion)} \to h_{ij}^{\mathrm{TT}} \text{ (spin-2 radiation).}} \tag{27}$$

This hierarchy is useful because it is harder to fool. It has controls. Scalar fields cannot fake holonomy. Vector connections cannot fake helicity 2. Pure gauge does not survive TT projection. A real emergence claim must pass all of those gates.

The program has not yet derived gravity from a microscopic substrate. It has identified the structure that such a derivation would have to reproduce. That is the useful step.

A Minimal algebraic witness

On a triangle with vertices $0, 1, 2$ and oriented edges $(01), (12), (20)$, a connection is

$$A = (A_{01}, A_{12}, A_{20}). \tag{28}$$

The loop holonomy is

$$H = A_{01} + A_{12} + A_{20}. \tag{29}$$

If $A_{ij} = \Lambda_j - \Lambda_i$, then

$$H = (\Lambda_1 - \Lambda_0) + (\Lambda_2 - \Lambda_1) + (\Lambda_0 - \Lambda_2) = 0. \tag{30}$$

But $A = (1, 1, 1)$ gives $H = 3$, so no global scalar clock potential can generate it.

B Fourier TT projector

For a nonzero wavevector k_i, define the transverse projector

$$P_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j. \tag{31}$$

The TT projection of a symmetric tensor h_{ij} is

$$h_{ij}^{\mathrm{TT}} = \left(P_i{}^a P_j{}^b - \frac{1}{2} P_{ij} P^{ab} \right) h_{ab}. \tag{32}$$

This projection removes trace and longitudinal content and leaves two degrees of freedom for each nonzero wavevector in three spatial dimensions.

C References

References

[1] S. Abramsky and A. Brandenburger. The sheaf-theoretic structure of non-locality and contextuality. *New Journal of Physics* 13, 113036 (2011). arXiv:1102.0264.

[2] S. Abramsky, S. Mansfield, and R. S. Barbosa. The cohomology of non-locality and contextuality. *Electronic Proceedings in Theoretical Computer Science* 95, 1–14 (2012). arXiv:1111.3620.

[3] M. Desbrun, A. N. Hirani, M. Leok, and J. E. Marsden. Discrete exterior calculus. arXiv:math/0508341 (2005).

[4] A. N. Hirani. *Discrete Exterior Calculus*. Ph.D. thesis, California Institute of Technology (2003).

[5] K. S. Yee. Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Transactions on Antennas and Propagation* 14(3), 302–307 (1966).

[6] M. L. Ruggiero and A. Tartaglia. Gravitomagnetic effects. arXiv:gr-qc/0207065 (2002).

[7] C. W. F. Everitt et al. Gravity Probe B: final results of a space experiment to test general relativity. *Physical Review Letters* 106, 221101 (2011).

[8] I. Ciufolini and E. C. Pavlis. A confirmation of the general relativistic prediction of the Lense-Thirring effect. *Nature* 431, 958–960 (2004).

[9] A. Tartaglia. From Kerr to Heisenberg: gravitational clock effects and gravitomagnetism. *Universe* 7(4), 86 (2021).

[10] K. J. Li, L. Iorio, and others. Generic gravito-magnetic clock effects. *Monthly Notices of the Royal Astronomical Society* 530(1), 1118–1131 (2024).

[11] M. Maggiore. *Gravitational Waves, Volume 1: Theory and Experiments*. Oxford University Press (2008).

[12] C. W. Misner, K. S. Thorne, and J. A. Wheeler. *Gravitation*. W. H. Freeman (1973).

[13] T. Regge. General relativity without coordinates. *Il Nuovo Cimento* 19, 558–571 (1961).