Condensed Timing Geometry

Observable Spacetime as a Sheaf of Clock-Connection Closures

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Abstract

This paper proposes *Condensed Timing Geometry*: a framework in which observable spacetime is not taken first as a manifold, a graph, or a scalar field, but as the stable global section of compatible timing observations over a category of finite relational probes. The central correction developed here is that scalar timing alone is too flat to carry geometry. A scalar load defines exact relative offsets, and exact offsets have zero loop holonomy. Curvature therefore cannot live in the scalar field itself. It lives in a clock-connection: an edge-valued timing 1-cochain whose loop holonomy obstructs global synchronization.

The framework begins with probe contexts and assigns to each context timing data, local synchronization charts, and possible clock-connection fields. A global observable world exists when these local timing charts glue. Nonzero holonomy gives a cohomological obstruction to gluing, analogous in form to the sheaf-theoretic obstruction to global sections in contextuality. In discrete exterior calculus language, scalar timing is the exact sector, topology is the harmonic sector, and radiating curvature belongs to the coexact connection sector.

First-pass toy diagnostics support the mathematical distinction. Scalar-gradient controls generate machine-zero curvature in the connection dynamics, while localized connection kicks produce finite-speed propagating curvature pulses with approximately linear long-wavelength dispersion. This is not yet a theory of gravity. The connection sector is Maxwell-like or spin-1-like; genuine gravity would require a further lift to a symmetric rank-2, spin-2 timing/metric field. The contribution of this paper is therefore narrower and more falsifiable: it identifies the correct geometric primitive as a timing connection and the first curvature observable as a cohomological obstruction to global clock synchronization.

Central thesis

Spacetime is not first a manifold. Observable spacetime begins as the stable global section of compatible timing observations over relational probes. Curvature appears when the clock-connection carries a nontrivial obstruction to global synchronization.

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1 Why the primitive has to change

A theory of emergent spacetime can fail because the equations are wrong. It can also fail because the primitive object is wrong.

The scalar-timing idea begins naturally. Suppose a structured substrate assigns a scalar load Λ_i to locations, nodes, or probe regions. Suppose local clock rates respond by

$$\omega_{\mathrm{eff},i} = \frac{\omega}{1+\alpha\Lambda_i}, \qquad \frac{\delta\omega_i}{\omega} \approx -\alpha\Lambda_i \quad (|\alpha\Lambda_i| \ll 1). \tag{1}$$

This produces a clean timing-first observable. It can model static redshift-like effects. It can be connected to graph Laplacians, screened Green functions, and effective dimensions. But it cannot by itself carry curvature in the stronger sense needed for geometry.

The reason is simple and decisive: a scalar field only produces exact relative offsets. If the relative timing offset on an oriented edge $i \to j$ is generated by a scalar potential,

$$A_{ij} = \Lambda_j - \Lambda_i, \tag{2}$$

then the sum around every closed loop vanishes. No loop fails to synchronize. No holonomy remains. No cohomological obstruction appears.

This forces the pivot of the program. The scalar field is not the geometric primitive. It is the exact, flat, potential-like sector of a larger object. The geometric primitive is a timing connection A: an assignment of relative clock offsets to oriented edges. The connection can have nonzero loop holonomy even when every local patch is internally consistent. That holonomy is the first curvature-like observable.

Thus the paper's object is not *scalar geometry*. It is *timing geometry*.

2 Probe contexts and timing observations

The framework is built from observations rather than from a pre-given space. Let Probe be a category of admissible probe contexts. An object $P \in \mathsf{Probe}$ consists of

$$P = (K_P, S_P, \mathcal{I}_P, \epsilon_P), \tag{3}$$

where K_P is a finite graph or 2-complex accessible to the probe, S_P is a set of clocks or readout channels, $\mathcal{I}_P$ is an allowed intervention class, and ϵ_P is the resolution/coarse-graining scale.

A morphism $f : Q \to P$ represents refinement, restriction, subcomplex inclusion, coarse graining, or a change of readout. This is deliberately chart-like. A probe context is not the world; it is one observational cut through which timing data are retained.

Define a timing observation assignment

$$\mathcal{T} : \mathsf{Probe}^{op} \longrightarrow \mathsf{Obs}, \tag{4}$$

where Obs is a category of timing data. For each probe P, the assignment may include

$$\mathcal{T}(P) = (K_P, A_P, F_P, \Lambda_P, \omega_{\mathrm{eff},P}, C_P)\,. \tag{5}$$

Here A_P is the timing connection on oriented edges, $F_P = \mathrm{d}A_P$ is its curvature on plaquettes or loops, Λ_P is the scalar/exact component, $\omega_{\mathrm{eff},P}$ is the clock readout, and C_P is a possible map from internal timing data to external sensors.

Definition 1 (Observable timing geometry)**.** *An observable timing geometry is an approximately gluable family of local timing sections over a probe cover, together with its residual obstructions to global synchronization.*

This definition avoids saying that the graph is space. The graph or complex is a chart. Geometry is the compatibility class of timing data across charts.

3 From scalar load to clock-connection

Let K be a finite oriented graph, or the 1-skeleton of a finite 2-complex. A scalar timing load is a 0-cochain

$$\Lambda \in C^0(K;\mathbb{R}). \tag{6}$$

A clock-connection is a 1-cochain

$$A \in C^1(K;\mathbb{R}), \tag{7}$$

assigning an antisymmetric offset to oriented edges:

$$A_{ji} = -A_{ij}. \tag{8}$$

The coboundary $\mathrm{d}_0 : C^0 \to C^1$ maps scalar loads to exact timing offsets:

$$A = \mathrm{d}_0\Lambda, \qquad A_{ij} = \Lambda_j - \Lambda_i. \tag{9}$$

If the complex has plaquettes, the next coboundary $\mathrm{d}_1 : C^1 \to C^2$ maps connections to curvature:

$$F = \mathrm{d}_1 A. \tag{10}$$

For a plaquette $p = (i, j, k, \ldots)$, this is the oriented loop sum

$$F_p = \sum_{e \in \partial p} \sigma(e,p) A_e. \tag{11}$$

Theorem 1 (Scalar timing is holonomy-free)**.** *If $A = \mathrm{d}_0\Lambda$ for a single-valued scalar Λ, then the holonomy around every closed cycle γ vanishes:*

$$\oint_\gamma A = 0. \tag{12}$$

Equivalently, $\mathrm{d}_1 A = 0$ *on every plaquette and* $[A] = 0$ *in first cohomology.*

Proof. For any oriented cycle $\gamma = (i_0, i_1, \ldots, i_n = i_0)$,

$$\oint_\gamma A = \sum_{m=0}^{n-1} (\Lambda_{i_{m+1}} - \Lambda_{i_m}) = \Lambda_{i_n} - \Lambda_{i_0} = 0. \tag{13}$$

The plaquette statement is the local version of the same identity, $\mathrm{d}_1\mathrm{d}_0 = 0$. Therefore exact timing offsets are cohomologically trivial. □

This theorem is the conceptual hinge. A scalar field may be important as a potential-like sector, but it cannot carry the loop obstruction that would make geometry nontrivial. To obtain curvature, one must promote timing from scalar load Λ to connection A.

4 Global sections, obstruction, and synchronization

Suppose a probe context P is covered by local timing charts $\{P_a \to P\}$. Each chart has local timing data $s_a \in \mathcal{T}(P_a)$. Pairwise consistency on overlaps asks whether

$$s_a|_{P_a \cap P_b} = s_b|_{P_a \cap P_b}. \tag{14}$$

But pairwise consistency is not enough. The central sheaf-theoretic question is whether the local sections extend to a global section $s \in \mathcal{T}(P)$ satisfying

$$s|_{P_a} = s_a \qquad \text{for every } a. \tag{15}$$

In Abramsky and Brandenburger's sheaf-theoretic account of contextuality, nonlocality and contextuality are characterized as obstructions to the existence of such global sections [1]. Cohomological refinements assign a Cech cohomology obstruction; nonvanishing is a sufficient witness of contextuality [2].

The timing-geometric analogue is direct. Local clocks may synchronize on every patch, and patches may agree on overlaps, while no single global clock assignment exists. The obstruction is loop holonomy.

Definition 2 (Clock-synchronization obstruction)**.** *For a clock-connection $A \in C^1(K;\mathbb{R})$, the synchronization obstruction on a cycle γ is*

$$\mathcal{H}_\gamma(A) = \oint_\gamma A. \tag{16}$$

The obstruction vanishes for all cycles if and only if A is globally synchronizable up to harmonic/topological qualifications determined by the complex and boundary conditions.

On a simply connected complex, vanishing loop holonomy implies A is exact. On spaces with nontrivial first cohomology, harmonic components can remain: flat but globally nontrivial timing twists. These do not radiate local curvature but encode topological synchronization offsets.

This is the corrected gluing principle:

Curvature is not pairwise mismatch. Curvature is obstruction to global synchronization.	(17)

A pairwise residual can be useful as a numerical sanity check, but it is not the invariant. The invariant is the cohomology class or the equivalent set of loop holonomies.

5 Discrete exterior calculus and the Hodge split

Discrete exterior calculus supplies the correct algebraic home for clock-connection geometry on graphs and complexes [3]. On a finite complex, cochains carry discrete exterior derivatives

$$C^0 \xrightarrow{\mathrm{d}_0} C^1 \xrightarrow{\mathrm{d}_1} C^2. \tag{18}$$

With inner products and adjoints, the 1-form Hodge Laplacian is

$$\Delta_1 = \mathrm{d}_0\mathrm{d}_0^* + \mathrm{d}_1^*\mathrm{d}_1, \tag{19}$$

up to convention on whether coboundary matrices are represented as rows or columns.

The corresponding Hodge decomposition is

$$C^1(K;\mathbb{R}) = \operatorname{im} \mathrm{d}_0 \;\oplus\; \mathcal{H}^1 \;\oplus\; \operatorname{im} \mathrm{d}_1^*, \tag{20}$$

where:

- $\operatorname{im} \mathrm{d}_0$ is the exact sector: scalar timing gradients;
- $\mathcal{H}^1 = \ker \Delta_1$ is the harmonic sector: topological holonomy;
- $\operatorname{im} \mathrm{d}_1^*$ is the coexact sector: curvature-carrying, radiative connection modes.

This decomposition gives the theory's new ladder:

$$\Lambda \quad \longrightarrow \quad A \quad \longrightarrow \quad F = \mathrm{d}A \quad \longrightarrow \quad h_{ij} \text{ or spin-2 completion.} \tag{21}$$

Scalar timing is the exact shadow. The clock-connection carries curvature. A further tensor lift would be required for gravity.

6 Connection dynamics and emergent light cones

The simplest timing-connection dynamics are wave dynamics on 1-forms:

$$\frac{\partial^2 A}{\partial t^2} + \Delta_1 A = J, \tag{22}$$

with a possible current/source term J. In source-free form, each eigenmode of Δ_1 with eigenvalue $\lambda_n \geq 0$ oscillates with

$$\omega_n = \sqrt{\lambda_n}. \tag{23}$$

The finite speed of propagation arises from locality of the discrete operator, just as finite-difference Maxwell/Yee schemes propagate electromagnetic waves on a lattice [5].

This is not a derivation of gravity. It is a clean way to separate three timing sectors:

- scalar-gradient kicks are exact and do not generate curvature;
- harmonic modes encode global topological clock offsets and have zero frequency in the idealized model;
- coexact connection modes propagate curvature.

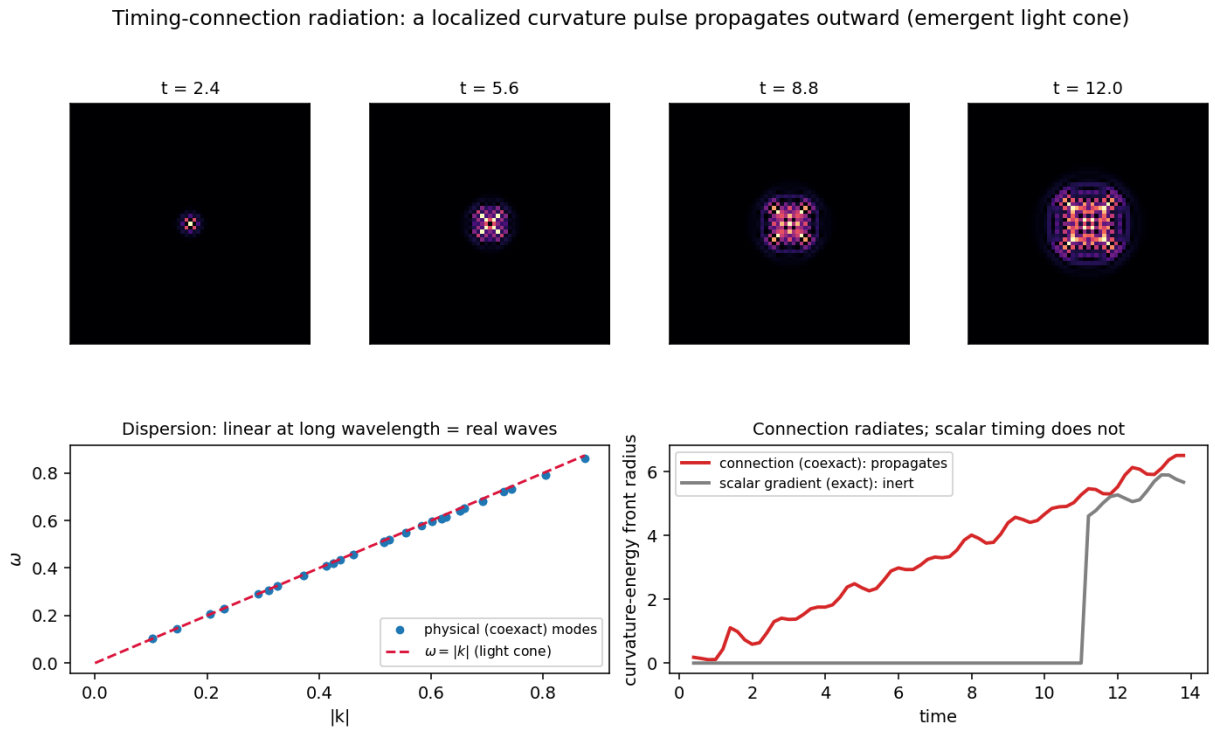

Figure 1: Toy timing-connection dynamics. A localized coexact curvature pulse propagates outward with an emergent light-cone-like envelope. The dispersion diagnostic is approximately linear at long wavelength, while a scalar-gradient control remains inert with respect to curvature radiation. This figure is an illustrative diagnostic of the connection model, not evidence of physical spacetime curvature.

In one finite torus diagnostic, the Hodge split separated the 1-form space into exact, coexact, and harmonic sectors, with the harmonic dimension matching the two independent torus cycles. The coexact sector displayed approximately linear long-wavelength dispersion, while scalar-gradient controls generated only numerical-roundoff curvature. The point is not the particular numbers. The point is structural: exact scalar timing and coexact connection timing are dynamically distinguishable.

7 The minimal holonomy witness

The smallest obstruction is a single loop. On a triangle or plaquette, choose local timing offsets that synchronize every local edge chart but whose oriented sum around the loop is nonzero. Locally, every chart is consistent. Globally, no clock potential exists.

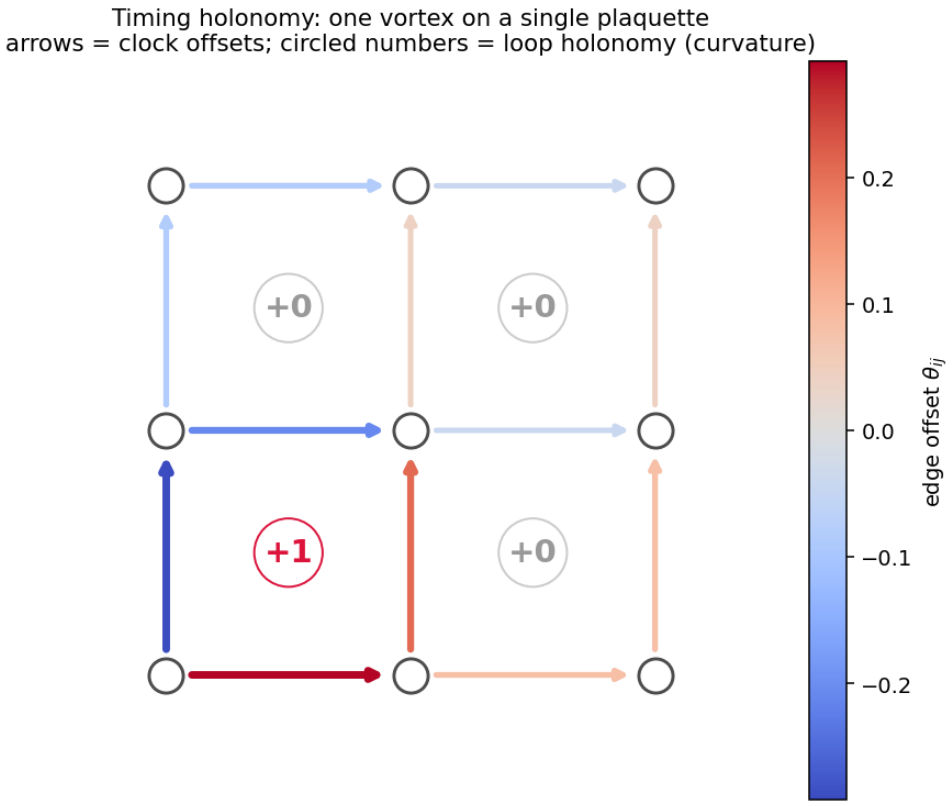

Figure 2: Toy timing holonomy on a plaquette lattice. Edge arrows represent relative clock offsets A_{ij}. Circled numbers indicate loop holonomy $F = \mathrm{d}A$. A single plaquette carries nonzero timing curvature while neighboring plaquettes remain flat. This is the discrete version of curvature as failure of global synchronization.

This witness is dimension-agnostic. It does not depend on recovering an effective dimension near three. It depends only on whether local timing data can be globally synchronized. That makes it a better primitive test than any scalar shell law.

8 What happens to scalar timing and effective dimension?

The scalar law in Eq. (1) is not discarded. It is demoted to the exact sector.

A scalar potential can still produce redshift-like timing gradients. A screened graph closure may still be useful:

$$(\kappa L_w + \mu^2 I)\Lambda = A\rho. \tag{24}$$

If a scalar Green response has an effective dimension $d_{\mathrm{eff}} \approx 3$, it can yield an inverse-radius profile,

$$\Lambda(r) \sim \frac{1}{r}, \qquad \frac{\delta\omega}{\omega} \sim -\frac{1}{r}. \tag{25}$$

That remains a meaningful static potential-like branch. But it is not the carrier of curvature, radiation, or nontrivial gluing.

The revised hierarchy is:

Layer	Mathematical object	Status
Scalar timing	0-cochain Λ	Potential-like, exact, holonomy-free
Relative clock offsets	1-cochain A	Clock-connection, can fail global synchronization
Curvature/holonomy	2-cochain $F = \mathrm{d}A$, cycle integrals	First nontrivial geometry-like observable
Radiation	Coexact dynamics under Δ_1	Maxwell-like, finite-speed connection waves
Gravity candidate	Symmetric tensor field h_{ij} or Regge-like metric perturbation	Not yet derived; required for spin-2

Table 1: Revised layer structure. Scalar timing is retained, but curvature begins only at the connection/holonomy layer.

This is a stricter and more defensible theory. It stops asking scalar timing to do a job that exact scalar fields cannot do.

9 Why this is not yet gravity

The connection model resembles lattice electromagnetism more than general relativity. A 1-form connection naturally supports Maxwell-like spin-1 propagation. Linearized gravitational waves, by contrast, are described by symmetric rank-2 metric perturbations h_{ij} with transverse-traceless spin-2 degrees of freedom.

Therefore the honest claim is:

> Timing connections can carry curvature-like holonomy and radiate, but they are not yet spin-2 gravity.

A gravitational extension would require a further lift, for example:

- a symmetric edge/face tensor field on a simplicial complex;
- linearized Regge calculus variables with gauge constraints;
- a discrete transverse-traceless projection and polarization test;
- a demonstration of spin-2 wave modes rather than vector connection modes.

The present paper should therefore be read as identifying the first nontrivial geometric primitive, not as completing emergent gravity.

10 Experimental and numerical program

The revised program has clearer tests than the scalar-only version.

10.1 Test 1: scalar exactness control

Generate timing offsets from scalar loads $A = \mathrm{d}\Lambda$. Verify that all loop holonomies vanish:

$$\oint_\gamma A = 0 \tag{26}$$

for every cycle γ, up to numerical or experimental noise. Any nonzero result in this control reveals error, hidden connection structure, or unmodeled physics.

10.2 Test 2: minimal holonomy witness

Construct a probe cover with locally synchronizable clock charts but nonzero total loop holonomy. Show that pairwise agreement can hold while no global clock assignment exists. This is the timing analogue of local consistency with global obstruction.

10.3 Test 3: Hodge sector decomposition

For a finite complex, decompose measured or simulated timing offsets into exact, harmonic, and coexact components:

$$A = A_{\mathrm{exact}} + A_{\mathrm{harm}} + A_{\mathrm{coexact}}. \tag{27}$$

Only the coexact component should carry local curvature radiation under Δ_1-type dynamics.

10.4 Test 4: propagation and dispersion

Excite a localized coexact timing pulse and estimate the front radius versus time. Fit long-wavelength dispersion:

$$\omega(k) \approx c_{\mathrm{eff}}|k|. \tag{28}$$

A finite c_{eff} is the connection-sector speed limit.

10.5 Test 5: externalization kernel

An internal clock-connection does not automatically imply an externally measurable field. The external map must be specified:

$$A_{\mathrm{ext}}(x) = \sum_e C(x, e)A_e, \tag{29}$$

with controls for distance, shielding, material substrate, resonances, and ordinary electromagnetic or thermal couplings.

10.6 Test 6: spin-2 lift

If the goal is gravity, the next numerical frontier is a symmetric tensor or Regge-like model. The minimum pass condition is not merely wave propagation. It is the emergence of transverse-traceless, two-polarization spin-2 modes.

11 Claim hierarchy

The claims of the framework should be kept separate.

Claim	Meaning	Status
Scalar exactness	Scalar timing offsets have zero holonomy	Theorem-level
Holonomy obstruction	Nonzero loop offset obstructs global synchronization	Theorem-level on finite complexes
Connection radiation	Coexact 1-form dynamics support finite-speed waves	Standard / toy-model
Physical external field	Internal connection couples to an external timing sensor	Experimental / open
Gravity	A spin-2 tensor sector emerges from the same substrate	Open; not established
Dimension selection	The observed 3D branch is dynamically selected	Open; not solved here

Table 2: Claim hierarchy. The paper's strongest established content is mathematical, not yet physical.

12 Conclusion

Condensed Timing Geometry replaces the scalar-only primitive with a clock-connection primitive.

The central result is a correction:

$$\text{scalar timing} \neq \text{geometry carrier}. \tag{30}$$

A scalar timing load is an exact 0-cochain potential. It can generate local redshift-like gradients, but it cannot produce loop holonomy, nontrivial gluing obstruction, or curvature radiation. The first genuine geometry-like object is the timing connection A, because it can obstruct global clock synchronization.

The revised thesis is therefore:

> Observable spacetime begins where local timing charts either glue into a global clock or fail by a measurable holonomy class.

This framework is useful precisely because it narrows the claim. It does not assert emergent gravity. It identifies the mathematical layer at which curvature first becomes possible. It gives a falsifiable

diagnostic: measure or compute the cohomological obstruction to global synchronization. It gives a dynamical next step: propagate the connection under a 1-form wave operator. And it names the remaining frontier honestly: the lift from spin-1-like timing connection to spin-2-like metric geometry.

If future work finds only exact scalar timing, the geometry is flat in the relevant cohomological sense. If it finds nonzero timing holonomy, then the first curvature observable has appeared. If that holonomy radiates with finite speed, the connection sector has dynamics. If a tensor lift yields spin-2 transverse-traceless modes, then the program can honestly begin to speak of gravity.

Until then, the right object is not scalar spacetime.

It is clock-connection geometry.

A Minimal algebraic witness

Let a triangle have vertices $0, 1, 2$ and oriented edges $(01), (12), (20)$. A connection is a triple

$$A = (A_{01}, A_{12}, A_{20}). \tag{31}$$

The loop holonomy is

$$H = A_{01} + A_{12} + A_{20}. \tag{32}$$

If A is exact, $A_{ij} = \Lambda_j - \Lambda_i$, then

$$H = (\Lambda_1 - \Lambda_0) + (\Lambda_2 - \Lambda_1) + (\Lambda_0 - \Lambda_2) = 0. \tag{33}$$

But $A = (1, 1, 1)$ has $H = 3$, so no scalar Λ can generate it. This is the smallest timing obstruction.

B Algorithmic sketch

Given a finite oriented 2-complex:

1. Build the incidence matrix D_0 from vertices to edges.
2. Build the incidence matrix D_1 from edges to faces.
3. Represent a timing connection as a vector $A \in \mathbb{R}^{|E|}$.
4. Compute curvature $F = D_1 A$.
5. Compute cycle holonomies using a cycle basis B: $H = BA$.
6. Compute the 1-form Laplacian $\Delta_1 = D_0 D_0^T + D_1^T D_1$, depending on orientation conventions.
7. Project A into exact, harmonic, and coexact components using least squares or eigenspace projectors.
8. Evolve $\ddot{A} + \Delta_1 A = 0$ for source-free connection waves.

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