CORRECTION AND SCOPE

The Scalar Shadow of Co-Emergent Geometry Clock Lag as the Minimal Readable Projection of a Relational Universe, with a Microscopic Bridge and Closure Hierarchy

The weighted-Laplacian mechanism is a result for the specified coupler model. Its proposed extension to a general scalar clock law remains open. Nine source/window repairs restored closure in zero simulator cases.

Manuscript · p. 1, unifying edition

A Test Protocol for Scalar Timing Closure and Dimensional Selection

Evidence · Scalar source/window residual audit

The Scalar Shadow of Co-Emergent Geometry

Clock Lag as the Minimal Readable Projection of a Relational Universe, with a Microscopic Bridge and Closure Hierarchy

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Independent Research
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April 2026

Abstract

This paper unifies two prior drafts of scalar timing geometry. From the first, it inherits a microscopic bridge in which a positive weighted graph Laplacian $L_w = B^T \mathrm{diag}(w_e) B$ arises by Schur reduction of fast edge degrees of freedom, together with a three-way distinction between static closure, dynamic rate closure, and clock-phase closure that fixes the diagnostic shape of finite-coupler failure. From the second, it inherits a sharper philosophical posture: clock lag is not chronologically prior to gravity; it is the minimal readable projection of a co-emergent relational structure. The two stances are not in tension. The genetic cascade "substrate $\to$ load $\to$ clock $\to$ gravity" was always a description of explanatory priority, not of temporal succession; the present synthesis makes that explicit.

The minimal scalar timing law, $\omega_{\mathrm{eff}} = \omega/(1 + \alpha\Lambda)$, is paired with operator closure $(\kappa L_w + \mu^2 I)\Lambda = A\rho$, dynamic completion through a finite fast scale Ω, and an explicit kernel $C(x, i)$ separating internal load from any externally measurable signal. The strong empirical test is a single scaling window in which a diffusion-based and a shell-profile-based estimator of effective dimension agree on $d_{\mathrm{eff}} \approx 3$, the timing law fits, and modal relaxation tracks the spectrum of L_w. Mass, magnetism, and tensor gravity are not denied; they are re-located as higher-readability projections of the same co-emergent structure. The result is not a completed theory of gravity but a disciplined research program whose failure modes have shape.

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1. Introduction: the danger of explanatory order

When a theory tries to explain time, it must be careful not to smuggle time into its explanation. It is tempting to say that relational updating comes first, then scalar load, then clock lag, then gravity. As a map of conceptual dependence this is useful. As a literal chronology it is incoherent: if time itself is emergent, "first" cannot mean "earlier in time" at the deepest level. A pre-geometric substrate does not wait, then make clocks, then wait again, then make gravity. Waiting is already a temporal concept.

The careful claim is the following:

> *The relational universe may cohere all at once, but finite instruments do not read it all at once.*

A clock does not measure the whole substrate. A local observer does not directly receive the full tensor, gauge, topological, and informational structure of the world. They receive a reduced set of stable observables: ticks, phases, delays, correlations, accelerations, and records.

Among these reduced observables, scalar timing is the simplest. This is not the claim that clock lag is physically older than gravity. It is the claim that clock lag is *operationally prior*: the lowest-order projection through which a richer co-emergent geometry becomes measurable.

A chord is not assembled note by note, yet an instrument may detect the bass first. White light is not born color by color, yet a prism separates it into components. A relational universe may not build scalar timing before gravity; rather, scalar timing may be the component revealed when the full structure is projected through the measurement basis of a local clock.

The earlier slogan, "the clock before gravity," is therefore sharpened:

> *The clock is not before gravity in time. The clock is the scalar face of co-emergent geometry.*

2. Genesis versus readability

The main correction is the distinction between *genetic order* and *readability order*. A genetic order says

$$A \to B \to C,$$

as though A happens, then B, then C. A readability order says

$$R_{\text{full}} \longrightarrow \Pi_{\text{obs}}(R_{\text{full}}),$$

where R_{full} is the full relational structure and Π_{obs} is the projection imposed by finite instruments, finite resolution, coarse graining, and stable records.

What "co-emergence" does and does not mean. The word *co-emergent* is heavier than it looks and is restricted here to two specific claims. First, the sectors are not chronologically ordered: there is no temporal succession in which scalar timing is built before tensor gravity. Second, the sectors are not derivable from each other in a definitional cascade: scalar load is not a stepping stone from which gravity is later constructed by adding ingredients. A stronger ontological claim — that all sectors must be metaphysically simultaneous in the substrate — is *not* asserted. The substrate may carry causal or update-ordering structure that is not yet temporal in the metric sense, and that ordering may differentiate sectors in ways finite observers do not directly see. The present framework is neutral on that deeper question; what it commits to is the conjunction of non-chronology and non-cascade.

Why scalar is the simplest readable projection. The readability hierarchy is now central, so the argument for scalar primacy must be made explicit rather than left implicit. A scalar load requires one number per locus and a single clock susceptibility α. A vector or gauge sector requires orientation, circulation, and either edge variables with phase or a notion of holonomy around closed loops. A tensor-gravity sector requires the joint behavior of clocks and rods, free-fall trajectories, universal coupling across matter species, Lorentz-compatible propagation, and either a spin-2 mode or a constraint structure that mimics one. The instrument inventory grows monotonically: clock alone → clock plus oriented loop → clock plus rod plus matter species plus radiation. Scalar timing sits at the bottom of this inventory.

This is why "first" here means *first to be read by a minimal instrument*, not first to exist.

3. Relational substrate and projected sectors

Let the deepest substrate be described abstractly as a lawful relational system rather than a differentiable manifold. Depending on the model, it may be represented as a graph, hypergraph, tensor network, causal set, quantum cellular automaton, or another structure of updates, adjacencies, memory, constraints, and correlations. The full microscopic state is denoted ρ_{micro}.

Observers do not access ρ_{micro} directly. Instead, they access stable projected sectors. The scalar timing sector is the image of a coarse-graining channel

$$\mathcal{E}_{\text{scalar}} : \rho_{\text{micro}} \longmapsto \{W, L_w, \Lambda, \omega_{\text{eff}}\},$$

where $W = (w_{ij})$ is an effective weighted adjacency, $L_w = D - W$ the weighted graph Laplacian, Λ the retained scalar load, and ω_{eff} the effective clock rate. This projection discards most of the microscopic state. It does not retain the full phase structure, all nonlocal correlations, all gauge-like loops, or the entire global topology. It keeps the part that survives into scalar timing measurements. The scalar sector is therefore not the whole world; it is the smallest world a clock can read.

4. Postulates of co-emergent scalar timing geometry

Postulate I (relational substrate). The deepest physical description is not assumed to be a smooth spacetime manifold. It is modeled as a lawful relational substrate of updates, adjacencies, constraints, memory, and correlations.

Postulate II (co-emergence). Scalar timing, mass-like persistence, gauge-like circulation, tensor geometry, and classical records do not arise in a temporal sequence and are not built from each other in a definitional cascade. They are mutually co-emergent projections of one relational structure, in the restricted sense fixed in Section 2.

Postulate III (asymmetric readability). Although sectors are co-emergent, they are not equally readable. Finite observers access reduced projections. The scalar timing projection is simpler than tensor reconstruction because it requires only a scalar load and a clock response, while richer sectors demand additional instruments and structure.

Postulate IV (scalar timing law). The minimal timing response is

$$\omega_{\text{eff}} = \frac{\omega}{1 + \alpha\Lambda}, \qquad \frac{\delta\omega}{\omega} \approx -\alpha\Lambda \quad \text{(weak load)}.$$

Postulate V (operator closure). The scalar load is not an arbitrary fitted field. It satisfies, at leading order,

$$\big(\kappa L_w + \mu^2 I\big)\Lambda = A\rho,$$

with dynamical completion

$$\partial_t \Lambda + \Gamma\big(\kappa L_w + \mu^2 I\big)\Lambda = \Gamma A\rho(t).$$

The same operator that controls response must also control smoothing, relaxation, and effective dimension.

Postulate VI (richer sectors require richer observables). Mass, magnetism, gauge structure, topology, and tensor gravity are not denied. They are higher-readability sectors, requiring persistence, oriented loops, holonomy, stress-response, tensorial degrees of freedom, and universal coupling beyond what a clock alone provides.

5. The clock as a projection device

A clock in this framework is not merely an object inside geometry; it is a projection device that converts inaccessible relational structure into accumulated phase. With bare frequency ω and local scalar load $\Lambda_i(t)$,

$$\omega_{\text{eff}}(i,t) = \frac{\omega}{1+\alpha\Lambda_i(t)}, \qquad \phi_i(T) = \int_0^T \omega_{\text{eff}}(i,t)\,dt.$$

For two clocks A and B,

$$\phi_A(T) - \phi_B(T) = \int_0^T \left[\frac{\omega}{1+\alpha\Lambda_A(t)} - \frac{\omega}{1+\alpha\Lambda_B(t)}\right] dt,$$

and in the weak-load regime

$$\phi_A(T) - \phi_B(T) \approx -\alpha\omega \int_0^T \big[\Lambda_A(t) - \Lambda_B(t)\big] dt.$$

This is the minimal measurable signature: not the full geometry, not the source, not the tensor structure — only differential accumulated phase. The clock does not tell us what the substrate is. It tells us that the substrate casts a scalar timing shadow.

6. Scalar load from an effective graph

The scalar sector is modeled on an effective weighted graph $\mathcal{G} = (V, E, w)$. Weights w_{ij} may arise from microscopic couplings, bond occupations, frustration patterns, dressed interactions, or another medium-specific extraction map. The weighted Laplacian is

$$(L_w f)(i) = \sum_j w_{ij}\big[f(i) - f(j)\big].$$

The leading scalar closure is the screened elliptic equation

$$(\kappa L_w + \mu^2 I)\Lambda = A\rho.$$

This is a leading weak-load closure, not a final ontology. Beyond it,

$$(\kappa L_w + \mu^2 I)\Lambda + \beta\Lambda^2 + \eta L_w^2\Lambda + \int_0^t M(t-s)\Lambda(s)\,ds = A\rho$$

collects the obvious next-order corrections: nonlinearity, higher-order graph operators, and memory. The scientific question is whether the leading closure suffices in any real or simulated scaling window.

7. A microscopic bridge: fast edge variables

The scalar timing program requires a mechanism that *produces* L_w rather than merely postulates it. Without such a bridge, Postulate V is an ansatz. A minimal fast-coupler model supplies the bridge.

Let Λ_i be slow node loads and x_e fast edge variables. Let B be the incidence matrix of an underlying substrate graph. Consider

$$\dot{\Lambda} = -\mu^2\Lambda - B^T G x + A\rho(t), \qquad \dot{x} = HB\Lambda - \Omega x.$$

Here G, H, Ω are edge-sector matrices. In the positive diagonal regime, the fast variables satisfy approximately

$$x \approx \Omega^{-1} H B \Lambda.$$

Substitution gives

$$\dot{\Lambda} \approx -\big(\mu^2 I + B^T G \Omega^{-1} H B\big)\Lambda + A\rho(t),$$

so that

$$\boxed{L_w = B^T G \Omega^{-1} H B.}$$

If $G\Omega^{-1}H = \mathrm{diag}(w_e)$ with $w_e > 0$, then

$$L_w = B^T \mathrm{diag}(w_e) B,$$

which is a positive weighted graph Laplacian. The substrate graph is specified, but the scalar response operator and its weights arise from microscopic edge parameters rather than from a node-level scalar ansatz. The Schur reduction is the place where "effective operator" acquires content.

8. Static, dynamic, and clock closure

The fast-coupler model yields three different kinds of closure, and conflating them is the source of most spurious agreement or disagreement with data.

8.1 Static closure

At steady state, eliminating x gives, exactly in the linear model,

$$(\mu^2 I + L_w)\Lambda = A\rho.$$

When $\mu = 0$, a connected graph carries a constant null mode; static closure then requires source neutrality, gauge fixing, or projection off the constant mode. This is the scalar analogue of removing an unobservable uniform offset.

8.2 Dynamic rate closure

For uniform fast scale Ω, each Laplacian eigenmode with eigenvalue λ_n obeys

$$(r + \mu^2)(r + \Omega) + \Omega\lambda_n = 0.$$

The Markovian scalar prediction is

$$\tau_n^{-1} \approx \mu^2 + \lambda_n.$$

This holds only when the fast sector is genuinely fast. Expanding,

$$r_{\mathrm{slow}} = -(\mu^2 + \lambda_n)\left[1 + \frac{\lambda_n}{\Omega} + O(\Omega^{-2})\right].$$

Rate errors scale as λ_n/Ω. High-λ modes fail first. The underdamped condition,

$$(\Omega - \mu^2)^2 < 4\Omega\lambda_n,$$

predicts memory and ringing in a specific window. This is not a blemish; it is the diagnostic shape that distinguishes the model from a generic fitting law.

8.3 Clock-phase closure

A local clock obeys

$$\dot{\phi}_i = \omega_0 + \alpha\Lambda_i(t),$$

so the accumulated shift is

$$\Delta\phi_i - \omega_0 t = \alpha \int \Lambda_i(t)\, dt.$$

A rate measurement probes poles of the response; a clock-phase measurement probes accumulated area. Under low-frequency protocols the clock integral can recover the static gain even when instantaneous rates are distorted by finite-coupler memory. Under high-frequency protocols, or when the fast sector is preloaded, the clock integral inherits the same finite-Ω errors.

Operational summary. Rate closure is the strict dynamic test. Clock closure is the robust low-frequency test. The same medium can pass a DC timing test while failing a high-frequency modal relaxation test, and the difference is not a contradiction — it is a measurement of Ω.

9. Effective dimension from the same operator

The operator L_w that governs scalar response also defines diffusion on the promoted graph,

$$K(t) = e^{-tL_w}, \qquad P_{\text{return}}(t) = \frac{1}{|V|} \operatorname{Tr} K(t).$$

A diffusion-based effective dimension is then

$$d_{\text{eff}}^{(\text{diff})}(t) = -2\, \frac{d \log P_{\text{return}}(t)}{d \log t}.$$

Independently, a scalar load sourced at a hotspot yields a shell-averaged profile

$$\Lambda_{\text{shell}}(r) = \frac{1}{|S_r|} \sum_{i \in S_r} \Lambda_i,$$

which in a Gaussian scaling regime obeys

$$\Lambda_{\text{shell}}(r) \sim r^{2-d_{\text{eff}}} \ \ (d_{\text{eff}} \neq 2), \qquad \Lambda_{\text{shell}}(r) \sim \log r \ \ (d_{\text{eff}} = 2).$$

The shell-based estimate is therefore

$$d_{\text{eff}}^{(\Lambda)}(r) = 2 - \frac{d \log |\Lambda_{\text{shell}}(r)|}{d \log r}.$$

The closure test is not whether either estimator gives a pleasing number. It is whether

$$d_{\text{eff}}^{(\text{diff})} \approx d_{\text{eff}}^{(\Lambda)}$$

in the same scaling window where the scalar timing law fits best.

10. The 3D branch and the first geometry

If the effective operator admits a Gaussian continuum scaling regime with $d_{\text{eff}} \approx 3$,

$$\Lambda(r) \sim \frac{1}{r}, \qquad \frac{\delta\omega}{\omega} \approx -\alpha\Lambda(r) \sim -\frac{1}{r}.$$

This resembles the radial envelope of weak-field gravitational redshift. The resemblance is not yet a derivation of gravity. It is a scalar precondition: in an effectively three-dimensional branch, scalar timing has the right radial form to become gravity-like if richer sectors are later supplied.

Caveat: $d_{\text{eff}} \approx 3$ is a target, not a result. The framework does not insert ordinary three-dimensional space at the bottom; it asks whether a promoted relational operator selects an effective dimension near three in the observable scalar branch. As written, this is a promissory note. Demonstrating the selection on at least one concrete substrate family — Poisson–Delaunay triangulations of a 3-manifold, cotangent Laplacians on irregular meshes, or another spectrally clean candidate — is the open empirical obligation. Until that demonstration is made, the section identifies the right test, not its result.

11. Substrate selection: which graphs become space

A relational universe cannot be an arbitrary graph if it is to look like a smooth world. Low-energy modes must be extended; dispersion must be approximately isotropic; locality must emerge rather than be assumed by name. Random geometric graphs can fail because their modes localize or scatter strongly. Poisson–Delaunay substrates and cotangent Laplacians are stronger candidates because they are connected to standard discrete approximations of continuum Laplace–Beltrami operators. The issue is not philosophical; it is spectral.

The minimal substrate-selection tests are:

1. Low eigenmodes extend across the substrate rather than localize.
2. $\lambda(k)$ scales as $|k|^2$ at low k.
3. Angular anisotropy of the dispersion falls with system size.
4. The scalar wave equation built from L_w admits approximately isotropic low-energy propagation.

This is not full Lorentz invariance. It is the spatial scalar-wave precondition for any Lorentz-compatible sector to follow.

12. Gravity as richer reconstruction, not later creation

Full gravity is not scalar timing. A scalar redshift-like channel is not equivalent to general relativity. A complete gravitational sector must account for a metric-like structure $g_{\mu\nu}$ or its effective equivalent, universal coupling, free-fall behavior, light cones, stress-energy response, Lorentz-compatible dynamics, gravitational radiation, and a spin-2-like sector or equivalent constraint structure.

Scalar timing relates to gravity as a shadow relates to an object. It is not the object; it is a projection of the object under a particular readout. The correct expression is therefore not

$$\Lambda \to \text{gravity},$$

but

$$R_{\text{full}} \longrightarrow \begin{cases} \Lambda,\ \omega_{\text{eff}} & \text{scalar timing projection,} \\ A_\mu,\ \text{holonomy} & \text{gauge-like projection,} \\ g_{\mu\nu} & \text{tensor reconstruction,} \\ \text{records} & \text{classical projection.} \end{cases}$$

Gravity and scalar timing are co-emergent projections of the same deeper structure. Scalar timing is not earlier; it is simpler to read.

13. Mass, magnetism, and records

Mass-like persistence. Mass may correspond to stable localized patterns — defects, knots, scars, standing excitations, self-maintaining concentrations of relational load. A clock may read their scalar effect before the full nature of their persistence is reconstructed.

Gauge-like circulation. Magnetism cannot be captured by scalar load alone. It requires orientation, phase, circulation, and holonomy. In graph language it lives on edges, cycles, and phase relations rather than nodes. Schematically:

$$\text{oriented information flow} \to \text{loop phase} \to \text{gauge field} \to \text{magnetism-like sector}.$$

Scalar timing asks how loaded a place is. Magnetism asks how relational phase circulates around a place.

Tensor gravity. Tensor gravity requires the joint reconstruction of metric-like structure from many observables — not just clock rate. It belongs to a richer projected algebra.

Classical records. A classical world requires stable, redundant, coarse-grained states that persist long enough to be compared. Records are not passive backdrop; they are part of the projection machinery itself.

14. Revised hierarchy: depth and readability

The earlier language of "before clock lag" and "after clock lag" is replaced by a hierarchy of projection depth. The table below is not a temporal sequence; it is a readability ladder. Lower levels are not earlier — they are less reconstructed. Higher levels are not later — they require more structure to read.

Level	Description	Status
0	Full relational substrate	Hidden / not directly observed
1	Stable relational topology	Adjacency, paths, loops, bottlenecks
2	Effective operator	W, L_w, response law
3	Scalar load	Λ, minimal retained scalar
4	Clock-readable projection	ω_{eff}, accumulated phase, lag
5	Gradient behavior	$\nabla\Lambda$, force-like scalar bias
6	Persistent excitations	Mass-like sources
7	Oriented loop sectors	Gauge / magnetism-like behavior
8	Tensor reconstruction	Gravity-like geometry
9	Stable records	Classical world

15. Internal loads and external observables

A scalar load *inside* a medium does not automatically imply an externally measurable scalar field. This distinction is essential and is one of the cleanest ways the program can fail.

Let Λ_i be internal scalar load on graph vertex i. An external sensor at position x measures, at most,

$$\Lambda_{\text{ext}}(x) = \sum_i C(x, i)\, \Lambda_i.$$

The kernel $C(x, i)$ determines how internal load couples to an external probe. Possible classes include:

- near-field or evanescent coupling, $C_{\mathrm{ev}}(x, i) \sim C_0\, e^{-d(x,i)/\ell_c}$;
- substrate-mediated diffusive coupling;
- radiative or boundary-modulated coupling;
- vacuum-like Green response;
- the null kernel, $C(x, i) \approx 0$.

An internal scalar timing sector can exist in a medium while producing no external geometric signal. That outcome would not make the internal theory meaningless, but it would limit the scope of any claim about external spacetime geometry. Reporting α, A, μ, Γ, and the kernel class together is the minimum honest output of an experiment.

16. Numerical and dynamical closure tests

The strongest numerical closure test combines static, dynamic, and dimensional checks in a single scaling window:

$$d_{\mathrm{eff}}^{(\mathrm{diff})} \overset{?}{\approx} d_{\mathrm{eff}}^{(\Lambda)} \overset{?}{\approx} 3, \qquad \omega_{\mathrm{eff}} = \frac{\omega}{1 + \alpha\Lambda} \text{ fits}, \qquad \tau_n^{-1} \approx \mu^2 + \lambda_n.$$

A meaningful pass requires:

1. a nontrivial scaling window in diffusion time or shell radius;
2. agreement between diffusion-based and shell-profile-based dimension estimates;
3. local minimization of timing-law residuals in the same window;
4. finite-size stability;
5. robustness to source placement and graph extraction thresholds;
6. dynamical relaxation rates tracking the Laplacian spectrum, with finite-Ω deviations scaling as λ_n/Ω and underdamped ringing in the predicted window.

If the timing law fits in one window while the dimension estimate works in another, strong closure fails. If the dimension estimate passes through three only transiently without a plateau, the result is suggestive but weak. If no Gaussian scaling window exists, the medium does not support the intended scalar branch. The virtue of the program is that failure has shape.

17. Experimental posture

The experimental program does not begin by claiming new gravity. It begins by bounding scalar timing response in controlled media. A minimal experiment requires:

1. a reconfigurable medium whose internal graph W can be changed;
2. a clock-like or phase-sensitive observable coupled to the medium;
3. topology changes at nearly fixed mass-energy distribution;
4. distance-law measurements;

5. thermal, electromagnetic, vibrational, and mechanical controls;
6. comparison of transient response against the graph Laplacian spectrum.

The desired signature is not "a frequency changed." Frequency changes are easy to fake through ordinary systematics. The desired signature is the conjunction

timing shift + graph dependence + topology sensitivity + modal relaxation + mass/force null controls.

A null result is useful: it constrains α, A, μ, Γ, or the coupling kernel $C(x, i)$, and tells us whether scalar timing is external, internal, screened, or absent in that medium.

18. Relation to existing ideas

The program sits near several established research families without coinciding with any of them. Wolfram-style hypergraph rewriting builds spacetime, relativity, and quantum behavior from abstract update rules; causal set theory replaces the continuum with a locally finite partial order; quantum graphity studies emergent locality and dimension from dynamical graphs; tensor-network and entanglement-based approaches encode geometry in quantum information; relational and Page–Wootters-type approaches treat time as a correlation between subsystems; scalar-gravity programs explored whether gravity could be a scalar field, and did not reproduce general relativity.

The present proposal differs by combining these motifs into a timing-first scalar-selection program: a relational substrate may co-emerge into many sectors at once, but the minimal readable projection for a local observer is scalar clock lag governed by an effective graph operator that also controls effective dimension and modal relaxation. The contribution is not a new microscopic rule. It is a synthesis with a sharp severe test.

19. Limits and proof levels

The word *proof* is used here only in restricted senses.

Proven inside the fast-coupler model. Under positive-diagonal edge-sector assumptions, Schur elimination of the fast variables produces a positive weighted graph Laplacian $L_w = B^T \mathrm{diag}(w_e) B$ and exact static scalar closure $(\mu^2 I + L_w)\Lambda = A\rho$.

Derived inside the linear dynamic model. Finite coupler speed produces mode-dependent corrections governed by $(r + \mu^2)(r + \Omega) + \Omega\lambda_n = 0$. The leading rate error scales as λ_n/Ω, with underdamped ringing in the predicted window.

Operationally motivated, not yet proven physically. Clock phase is a privileged observable because it integrates scalar load. This explains why timing may be robust in low-frequency protocols but does not prove universal clock protection.

Not yet proven. The paper does not prove mass, electromagnetism, full Lorentz invariance, external scalar geometry, general relativity, a spin-2 sector, or a theory of everything. The chief open empirical obligation is to demonstrate $d_{\mathrm{eff}} \approx 3$ *selection* on a concrete substrate family rather than imposition by construction. The chief open theoretical obligation is to show how the scalar projection extends to vector, gauge, and tensor projections without smuggling in the structure they are meant to explain.

The main risks of failure, which the program is designed to expose rather than hide:

1. The scalar timing law is only a fitting artifact.
2. The effective dimension does not stabilize in any scaling window.
3. The graph Laplacian fails to control timing and response together.
4. The external kernel $C(x, i)$ is null in accessible media.
5. Ordinary thermal or electromagnetic effects dominate the signal.
6. Tensor reconstruction does not follow from scalar closure.
7. The $d_{\text{eff}} \approx 3$ branch is imposed by model construction rather than dynamically selected.

These are not peripheral concerns. They are the tests.

20. Conclusion: the scalar face of a simultaneous world

The universe may not unfold according to the order in which we explain it. Clock lag need not happen before gravity. Scalar timing need not be built before geometry. Mass, gauge structure, tensor curvature, and records need not wait in line. They may be co-emergent aspects of a single relational structure, in the restricted sense that they are neither chronologically ordered nor cascaded by definition.

But observers do not receive that structure whole. They ask partial questions, use finite instruments, and extract stable projections. Among those projections, clock timing is unusually primitive: a local system accumulates phase, and phase can be compared.

The claim of scalar timing geometry is therefore not that the clock comes before gravity in time. It is that the clock may reveal the scalar face of co-emergent geometry before richer reconstruction is possible — and that the same operator L_w which governs that scalar face must also govern effective dimension, modal relaxation, and the diagnostic shape of finite-coupler failure.

A relational universe may be a chord. Gravity may be the full harmony. Clock lag may be the first note our instruments can reliably hear. The scientific task is to determine, in a single scaling window with severe controls, whether that note is real.

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