CORRECTION AND SCOPE

A Test Protocol for Scalar Timing Closure and Dimensional Selection

Original title · Numerical Closure Tests for Scalar Projection and Dimensional Selection

A protocol for comparing scalar timing observables. The subsequent residual audit tested nine source/window repairs and restored closure in zero cases.

Manuscript · p. 1, purpose

Evidence · Scalar source/window residual audit

Numerical Closure Tests for Scalar Projection and Dimensional Selection

Purpose

This paper defines the numerical closure tests that would move the scalar-selection program from coherent framework to internally tested result. The central aim is to compare two independent estimates of effective dimension in the same scaling window while simultaneously checking whether the scalar timing law fits best there.

1 Central Closure Question

The core hypothesis can be written as

$$H_{\text{micro}} \to W \to L_w \to d_{\text{eff}} \to G_{d_{\text{eff}}}(r) \to \Lambda(r) \to \omega_{\text{eff}}. \tag{1}$$

The strongest numerical closure test is

$$d_{\text{eff}}^{(\text{diff})} \overset{?}{\approx} d_{\text{eff}}^{(\Lambda)} \overset{?}{\approx} 3 \tag{2}$$

in the same scaling window where

$$\omega_{\text{eff}} = \frac{\omega}{1 + \alpha\Lambda} \tag{3}$$

fits best.

2 Promoted Graph Input

For a given microscopic state or medium instance, define the weighted graph by

$$W = (w_{ij}), \qquad D_{ii} = \sum_j w_{ij}, \qquad L_w = D - W. \tag{4}$$

The weights may come from bond occupation, dressed couplings, frustration indicators, or another medium-specific map. The closure test does not assume the microscopic origin; it assumes only that a promoted weighted graph can be extracted.

3 Two Independent Dimension Estimators

3.1 Diffusion-based estimator

Let

$$K(t) = e^{-tL_w}, \qquad P_{\text{ret}}(t) = \frac{1}{|V|} \operatorname{Tr} K(t). \tag{5}$$

Define the diffusion-based effective dimension by

$$d_{\mathrm{eff}}^{(\mathrm{diff})}(t) = -2\frac{d\log P_{\mathrm{ret}}(t)}{d\log t}. \tag{6}$$

A meaningful result requires an intermediate time interval over which this slope is approximately constant.

3.2 Shell-profile estimator

Around a chosen source or hotspot, define graph shells S_r and a shell-averaged scalar load

$$\Lambda_{\mathrm{shell}}(r) = \frac{1}{|S_r|}\sum_{i\in S_r}\Lambda_i. \tag{7}$$

In the Gaussian regime the asymptotic profile is

$$\Lambda_{\mathrm{shell}}(r) \sim \begin{cases} Ar^{2-d_{\mathrm{eff}}}, & d_{\mathrm{eff}} \neq 2, \\ A\log r, & d_{\mathrm{eff}} = 2. \end{cases} \tag{8}$$

Hence define the shell estimator on the noncritical branch by

$$d_{\mathrm{eff}}^{(\Lambda)}(r) = 2 - \frac{d\log|\Lambda_{\mathrm{shell}}(r)|}{d\log r}. \tag{9}$$

4 Timing-Law Fit

The scalar timing law predicts

$$\omega_{\mathrm{eff}} = \frac{\omega}{1+\alpha\Lambda}, \qquad \Delta y \equiv \frac{\delta\omega}{\omega} \approx -\alpha\Lambda \quad (|\alpha\Lambda| \ll 1). \tag{10}$$

For each candidate scaling window, fit α and evaluate the residual statistics of the full rational form and the weak-load linearization. The same window that supports the dimension estimators should also minimize timing-law residuals if the closure is real.

5 Static Closure Test

The scalar load itself should satisfy the screened graph closure

$$(\kappa L_w + \mu^2 I)\Lambda = A\rho. \tag{11}$$

Given ρ and measured/extracted Λ, one can fit (κ, μ, A) by least squares or Bayesian inversion and then compare the inferred Λ against data. This is a third internal consistency check, although it is less independent because it uses the same scalar field being fit.

6 Dynamical Closure Test

If time-resolved data are available, test the dynamical completion

$$\partial_t \Lambda + \Gamma(\kappa L_w + \mu^2 I)\Lambda = \Gamma A \rho(t). \tag{12}$$

With

$$L_w \phi_n = \lambda_n \phi_n, \tag{13}$$

mode decay rates should follow

$$\tau_n^{-1} = \Gamma(\kappa \lambda_n + \mu^2). \tag{14}$$

A measured relaxation spectrum that tracks the graph-Laplacian spectrum is a powerful discriminant against generic thermal or instrumental transients.

7 Finite-Size and Window Criteria

An apparent $d_{\text{eff}} \approx 3$ crossing is not enough. A genuine plateau should satisfy all of the following:

1. It persists over a nontrivial intermediate window in both diffusion time and shell radius.
2. The two independent estimators agree within uncertainty in that same window.
3. The timing-law residual is locally minimized in that same window.
4. The inferred value drifts more slowly as system size grows, not more rapidly.
5. The result is robust to moderate changes in source placement and graph extraction threshold.

A transient pass through 3 without these properties should be treated as suggestive only.

8 Failure Modes

The main ways closure can fail are:

- heavy-tailed or strongly anisotropic weight distributions,
- bottlenecks or near-tree sparsity destroying a clean Gaussian regime,
- shortcut-dominated graphs producing multiple scaling windows,
- source-placement dependence masquerading as a dimension plateau,
- timing-law fit improving in a window different from the dimension plateau,
- memory effects large enough that the static or Markovian closure is inadequate.

Failure is informative. It means either the chosen medium does not realize the scalar-selection program or the extraction map from microscopic state to promoted graph is inadequate.

9 Minimal Numerical Workflow

1. Extract or construct the promoted weighted graph W.
2. Compute L_w and the heat kernel return probability.
3. Estimate $d_{\mathrm{eff}}^{(\mathrm{diff})}(t)$.
4. Solve or measure the scalar field Λ and form shell averages.
5. Estimate $d_{\mathrm{eff}}^{(\Lambda)}(r)$.
6. Fit the timing law and record residuals across windows.
7. Fit the static closure parameters (κ, μ, A).
8. If time data exist, fit the dynamical spectrum and compare with λ_n.
9. Repeat over system sizes, topologies, and extraction thresholds.

10 Outcome Table

Numerical outcome	Interpretation
$d_{\mathrm{eff}}^{(\mathrm{diff})} \approx d_{\mathrm{eff}}^{(\Lambda)} \approx 3$ and timing law fits best there	Strong internal support for the scalar-selection program
Dimension estimators agree but not near 3	Scalar closure may exist, but the selected branch is not Newton-like
Timing law fits but dimension estimators disagree	Scalar phenomenology exists but the dimensional-selection interpretation is incomplete
No clean Gaussian window	Chosen medium or extraction map does not support the intended regime
Dynamical spectrum tracks λ_n	Strong evidence for graph-mediated scalar dynamics
Dynamical spectrum does not track λ_n	Memory, thermal, or nonlocal corrections dominate

11 Why This Paper Matters

The scalar-selection program is no longer short of conceptual material. What it needs is closure under its own rules. This note defines the minimal numerical criteria for that closure. Until these tests are run, the framework remains coherent and suggestive. If they pass, the program becomes substantially stronger. If they fail, the program becomes narrower but more truthful.