CORRECTED

Emergent Physical Laws from Self-Referential Quantum-Information Dynamics (SRQID): A Linear, Measurement-Free Hamiltonian with Lieb-Robinson Bounds

The locality bound in §3 stands. On a bounded-degree graph, uniformly bounded finite-range interactions limit how fast a disturbance can spread. Two numerical claims on page 5 change.

Propagation speed · p. 5, §§6–7

Original claim The paper reports 2.071 as the Lieb–Robinson velocity on eight sites.

Correction The fit used the first time a chosen commutator crossed 0.001 at six distances. Its slope, 2.071 sites per unit time, describes that threshold. The theorem gives an upper bound for all local observables. This fit does not determine it.

The far spin after a flip · p. 5, §§6–7

Original claim The far spin's Z component changed by at most 3.44×10^{-15}. The paper cited this as a no-signalling test and reported the energy drift of the same trajectory.

Correction A code error gave every amplitude in the measured state the same phase, freezing both readings. With the phases fixed, Z still barely changes. At time 3, X changes by -5.30×10^{-6} and the far spin's trace distance is 2.65×10^{-6}. The remote states agree at time zero and then diverge slightly, as the locality bound permits.

Manuscript · August 20, 2025 · §3, p. 3 and §§6–7, p. 5

Evidence · Eight-site spin check / Crossref deposit · SRQID on SSRN

Emergent Physical Laws from Self Referential Quantum Information Dynamics (SRQID): A Linear, Measurement Free Hamiltonian with Lieb Robinson Bounds

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Abstract

We present SRQID, a strictly *linear*, *time independent*, and *measurement free* Hamiltonian framework in which matter qubits live on graph vertices and finite dimensional bond registers live on edges. All updates are generated by a fixed, bounded degree, local Hamiltonian; there is no state dependent generator and no projective measurement. Locality implies a finite Lieb Robinson (LR) velocity that defines an emergent light cone. We provide exact structural statements and minimal numerics: commutator growth extraction of the LR velocity, a no signalling local quench test, and energy drift bounds. A companion paper (QATNU) develops a circuit realization (QCA+promotion) and phenomenology; here we supply the linear foundation it cites [10]. *Code and minimal outputs:* Harvard Dataverse (doi:10.7910/DVN/YZE8RI).

1 Introduction

A recurring challenge for emergent geometry proposals is the use of non linear or measurement based updates. SRQID is built to avoid both: the generator is a *fixed, local, time independent* Hamiltonian on a bounded degree graph; apparent "selection" or "promotion" phenomena arise from coherent dynamics in an *enlarged Hilbert space* with isometric ladders on bonds, not from projections.

Degrees of freedom. Let $G = (V, E)$ be a bounded degree simple graph. Each $i \in V$ hosts a two level matter system with Pauli operators $\{X_i, Y_i, Z_i\}$. Each edge $e = \langle i, j \rangle \in E$ hosts a finite dimensional bond register with Pauli like operators $\{\tau_{ij}^{x,y,z}\}$ acting on the lowest nontrivial subspace; higher levels are accessed via an isometric ladder (below).

Conventions. We set $\hbar = 1$ and use operator norms throughout.

2 Hamiltonian model

We consider the following time independent Hamiltonian on $\mathcal{H} = \bigotimes_{i\in V} \mathbb{C}^2 \otimes \bigotimes_{\langle i,j\rangle\in E} \mathbb{C}^{\chi_{ij}}$:

$$H \;=\; H_{\mathrm{mat}} + H_{\mathrm{bond}} + H_{\mathrm{int}}, \tag{1}$$

with (one convenient choice)

$$H_{\mathrm{mat}} = \frac{\omega}{2} \sum_{i\in V} X_i, \tag{2}$$

$$H_{\mathrm{bond}} = \sum_{\langle i,j\rangle\in E} \Big(J_0\, Z_i Z_j + \Delta_b\, n_{ij} \Big), \tag{3}$$

$$H_{\mathrm{int}} = g \sum_{\langle i,j\rangle\in E} F_{ij}\, \tau^x_{ij} \;+\; \kappa \sum_{\langle i,j\rangle\in E} C_{ij} \;+\; \lambda \sum_{\langle i,j\rangle\in E} F_{ij} \otimes (R_{ij} + R_{ij}^\dagger), \tag{4}$$

where:

- F_{ij} and C_{ij} are *fixed, local* operators (constructed from short Pauli strings) that (i) respond to frustration/entanglement locally and (ii) softly enforce bounded degree (no state dependence). A concrete tile is $F_{ij} = \frac{1}{2}(\mathbb{I} - Z_i Z_j)$ and C_{ij} a diagonal penalty that *disfavors* simultaneous activations beyond a chosen local bound.
- $n_{ij} = \sum_{m\geq 0} m\, \Pi^{(m)}_{ij}$ is the rung (bond-occupancy) operator on edge $\langle i,j\rangle$ with spectral projectors $\{\Pi^{(m)}_{ij}\}_{m=0}^{\chi_{ij}-1}$.
- $R_{ij} = \sum_{m=0}^{\chi_{ij}-2} |m{+}1\rangle\langle m|$ is the local ladder-raising isometry on the bond register (finite truncation). The last term in (4) provides an *explicit Hamiltonian mechanism* for coherent promotion $m \to m \pm 1$ when the local frustration tile F_{ij} is active. No projectors or state-conditioned generators are introduced; the enlarged Hilbert space and all couplings are present from $t = 0$.

Local degree and active-rung operators. Define the operator degree at vertex i by

$$d_i := \sum_{j\in\partial i} \sum_{m\geq 1} \Pi^{(m)}_{ij}, \qquad \partial i := \{j : \langle i,j\rangle \in E\}.$$

A convenient bounded penalty is $C_{ij} := (d_i - k_0)^2 + (d_j - k_0)^2$, which is diagonal and preserves linearity. We realize "active-rung" action by

$$\tau^\alpha_{ij} := \sum_m \Pi^{(m)}_{ij}\, \tau^{\alpha,(m)}_{ij}\, \Pi^{(m)}_{ij}, \qquad \alpha \in \{x, z\},$$

so τ^α_{ij} is block-diagonal and strictly local.

Isometric ladders *vs.* measurements. The Stinespring/Choi picture [5, 6] tells us any isometry can be realized unitarily on an enlarged space without projective measurement. Here the ladder is built directly into H via n_{ij} and R_{ij}; Eq. (1) is time independent and linear on $\mathcal{H}$ at all times.

Entanglement proxy (for plots only). For diagnostics we use the standard two site purity proxy $\Delta_{ij} = 1 - \operatorname{tr}\rho_i^2 - \operatorname{tr}\rho_j^2 + \operatorname{tr}\rho_{ij}^2 \in [0,1]$ computed *from trajectories*. The Hamiltonian does not depend on Δ_{ij}.

Clarification (penalties $\neq$ measurements). C_{ij} denotes a Hamiltonian penalty (diagonal operator), not a measurement; the generator in Eq. (1) is state-independent and linear throughout.

3 Locality and Lieb Robinson bound

Bounded-degree LR cone (standard). For finite-range, bounded-norm interactions on a bounded-degree graph, LiebRobinson (LR) bounds guarantee an emergent causal cone [1–3]. Our H satisfies these hypotheses by construction; thus there exist $v_{\mathrm{LR}}, \xi, c > 0$ such that for any $X, Y \subset V$ and local observables A_X, B_Y,

$$\big\|[A_X(t), B_Y]\big\| \;\le\; c\,\|A_X\|\,\|B_Y\|\,\exp\!\left(-\frac{\operatorname{dist}(X,Y) - v_{\mathrm{LR}}|t|}{\xi}\right). \tag{5}$$

An explicit conservative constant. Let $H = \sum_{X \subset V} h_X$ with $\operatorname{diam}(X) \le R$ and define

$$J_* := \sup_{x \in V} \sum_{X \ni x} \|h_X\|, \qquad z := \sup_{x \in V} |\partial x| \text{ (max degree)}. \tag{6}$$

Then for any $\mu > 0$ one has an LR bound of the form (5) with velocity

$$v_{\mathrm{LR}}(\mu) \;\le\; \frac{2}{\mu}\, C_\mu, \quad C_\mu := \sup_{x \in V} \sum_{X \ni x} \|h_X\|\,|X|\, e^{\mu \operatorname{diam}(X)} \;\le\; J_*(1+z)\, e^{\mu R}. \tag{7}$$

Optimizing over μ yields an explicit $v_{\mathrm{LR}} \lesssim 2e\,R\,J_*(1+z)$, which in our case (nearest-neighbor, R=1) depends only on $\omega, J_0, \Delta_b, g, \lambda, \kappa$, the operator norms $\|F\|, \|\tau^{x,z}\|$, and the degree cap z.[1]

4 Exact structural statements

- **Linearity and measurement free evolution:** The generator Eq. (1) is time independent and does not condition on the quantum state.
- **Bounded degree (soft cap):** The C_{ij} terms penalize configurations that would exceed a chosen local degree.
- **Isometry and unitarity:** The ladder is realized Hamiltonianly via n_{ij} and R_{ij}; evolution remains unitary on the enlarged Hilbert space.

[1] This is a standard Hastings–Koma/Nachtergaele–Sims constant; we state it here for parameter transparency. See also [2,3].

5 Mathematical foundations: self-adjointness and growth bounds

Self-adjointness and thermodynamic limit.

Lemma 1 (Essential self-adjointness and unitary dynamics)**.** *For finite-range, bounded-norm interactions on a bounded-degree graph, H defined by (1)–(4) is essentially self-adjoint on the local algebra. The dynamics $U(t) = e^{-itH}$ is a strongly continuous one-parameter unitary group on the quasi-local algebra, and satisfies an LR bound of the form (5).*

Proof sketch. Decompose $H = \sum_X h_X$ with $\sup_X \|h_X\| < \infty$ and $\mathrm{diam}(X) \le R < \infty$. Standard constructions (e.g., [3]) show the thermodynamic limit of local Hamiltonians exists, is essentially self-adjoint on the local observable algebra, and generates a strongly continuous dynamics obeying an LR bound. □

Degree growth bound and no runaway connectivity.

Lemma 2 (Local degree growth is LR-limited)**.** *Let $d_i = \sum_{j\in\partial i} \sum_{m\ge 1} \Pi_{ij}^{(m)}$. Then*

$$\frac{d}{dt}\langle d_i(t)\rangle \;=\; i\,\langle [H, d_i](t)\rangle \quad\Rightarrow\quad \left|\frac{d}{dt}\langle d_i(t)\rangle\right| \;\le\; \|[H, d_i]\| \;\le\; \alpha, \tag{8}$$

with α depending only on local operator norms and the degree cap z. Moreover, for any region X and its complement at distance r, $\langle d_i(t)\rangle$ outside the LR cone differs from its unperturbed value by at most $O(e^{-(r-v_{\mathrm{LR}}t)/\xi})$.

Proof sketch. Only terms in H supported on edges incident to i fail to commute with d_i, yielding a uniform bound $\|[H, d_i]\| \le c_0(\|F\|(|g|+|\lambda|) + |\kappa|\,\|C\|)$ with c_0 depending on z. The LR bound then controls the spread of any difference outside the cone. □

Promotion speed ≤ LR speed.

Proposition 1 (LR-limited promotion)**.** *For every edge $\langle i, j\rangle$ and rung projectors $\{\Pi_{ij}^{(m)}\}$, the expectation $\sum_m m\,\langle \Pi_{ij}^{(m)}(t)\rangle$ changes with a finite velocity bounded by the LR constants. In particular, a local quench supported in X alters $\langle \Pi_{ij}^{(m)}(t)\rangle$ at distance r by at most $O(e^{-(r-v_{\mathrm{LR}}t)/\xi})$.*

Proof sketch. Apply (5) to the commutator $[\Pi_{ij}^{(m)}(t), B_Y]$ with B_Y supported in the quench region $Y = X$. The Duhamel expansion and Grönwall inequality give the stated exponential suppression outside the cone. □

Remark (Clifford equivalence of stabilizer tiles). The concrete choice $F_{ij} = \frac{1}{2}(\mathbb{I} - Z_iZ_j)$ is without loss: alternative tiles used in the QATNU companion (e.g., Y_iY_j, or short stabilizer strings) are related by local Clifford rotations, leaving locality and LR constants invariant (up to norm factors).

6 Validated Numerics (2025 08 12)

Minimal, checkable numerics supporting the SRQID claims (details and scripts in the companion notebook; environment pinned in `requirements_qatn.txt`).

- **Lieb Robinson velocity:** From commutator growth thresholds ($\varepsilon = 10^{-3}$) we extract $v_{\mathrm{LR}} \approx 2.071$ on $N = 8$.
- **No signalling local quench:** Flipping site 0 at $t = 0$ changes the far end expectation by $\max_t |\Delta\langle Z_r(t)\rangle| \approx 3.44 \times 10^{-15}$ at $r = N-1$ (sub cone propagation).
- **Energy conservation:** For the time independent H, the energy drift is $\Delta E \approx 5.33 \times 10^{-15}$ across the evolution window.
- **Circuit cross check (reported from QATNU):** a separate layer unitarity test gives $\|U^\dagger U - I\|_2 = 1.83 \times 10^{-15}$.

Numerical parameters and integrator details

All specific scalars used for the plots ($\omega, J_0, \Delta_b, g, \lambda, \kappa, k_0$), lattice size/boundaries, time window, and tolerance are *emitted at run time* to `outputs/summary.txt` by `make run`. The commutator norms and quenches are computed by exact diagonalization (dense eigensolver, double precision); time evolution uses the spectral decomposition e^{-itH}, and reported energy drift is at machine precision. Re-running the default configuration in the Dataverse bundle reproduces the numbers above.

7 Methods (numerical)

Commutator growth. We evaluate $\|[Z_0(t), Z_r]\|$ by exact diagonalization for $N \leq 8$, record first passage times $t_{\mathrm{hit}}(r; \varepsilon)$ at threshold ε, and fit $r \approx v_{\mathrm{LR}}\, t + b$.

No signalling quench. Starting from $|0\rangle^{\otimes N}$ we compare $\langle Z_r(t)\rangle$ with and without an X at site 0 at $t = 0$; the difference remains below numerical noise until the cone arrival, consistent with (5).

Energy drift. We compute $E(t) = \langle H(t)\rangle$ along trajectories and report $\max_t E - \min_t E$.

Reproducibility. All scripts and the pinned environment file (`requirements_qatn.txt`) are provided as arXiv ancillary files and archived on Harvard Dataverse (doi:10.7910/DVN/YZE8RI). Running `make run` reproduces the LR-velocity fit, quench test, and energy-drift report.

8 Discussion

SRQID supplies a measurement free linear backbone with a controlled causal structure. It is intentionally conservative: we do not claim gauge cascades or spin 2 fits here. Those appear in QATNU and cite this paper for linearity and LR bounds [10]. Extensions include larger N tensor network simulations, disorder robustness, and bounds on effective cone renormalization under ladder depth.

Data and code availability

The reproducibility bundle (Makefile, pinned requirements, and scripts for SRQID and QATNU) is archived on Harvard Dataverse at doi:10.7910/DVN/YZE8RI. A mirror of the scripts is included as arXiv ancillary files.

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